

# Contents

<b>Riassunto</b>	<b>iii</b>
<b>Introduction</b>	<b>1</b>
<b>1 Preliminaries and auxiliary results</b>	<b>13</b>
1.1 Recall on semigroups theory . . . . .	13
1.2 Sectorial operators . . . . .	14
1.2.1 Perturbation of sectorial operators . . . . .	17
1.3 Analytic semigroups and spaces $D_A(\theta, p)$ . . . . .	18
1.4 Preliminaries of measure theory . . . . .	20
1.4.1 Weak convergence of measures . . . . .	22
1.4.2 Differentiation of measures . . . . .	23
1.4.3 Hausdorff measures and rectifiable sets . . . . .	24
1.5 Some further preliminaries . . . . .	24
<b>2 Generation of analytic semigroups by elliptic operators</b>	<b>29</b>
2.1 Assumptions and formulation of the boundary value problem . . . . .	29
2.2 Basic estimates for elliptic equations . . . . .	31
2.2.1 Analytic semigroups in $L^p(\mathbf{R}^n)$ , $1 < p < \infty$ . . . . .	33
2.2.2 $L^p$ -estimates on domains . . . . .	38
2.3 Generation of analytic semigroup in $L^\infty(\Omega)$ and in the space $C(\overline{\Omega})$ . . . . .	44
2.4 Elliptic boundary value problems in some Sobolev spaces of negative order	50
2.4.1 Formally adjoint boundary value problems . . . . .	51

2.5	Generation of analytic semigroups in $L^1(\Omega)$ . . . . .	58
<b>3</b>	<b>Estimates of the derivatives of solution of parabolic problems in <math>L^1(\Omega)</math></b>	<b>63</b>
3.0.1	Estimates of first order derivatives . . . . .	63
3.1	Estimates of second order derivatives . . . . .	65
3.1.1	Characterization of interpolation spaces between $D(A_1)$ and $L^1(\Omega)$	69
<b>4</b>	<b><math>BV</math> functions and parabolic problems: the first characterization</b>	<b>73</b>
4.1	The space $BV$ : definitions and preliminary results . . . . .	73
4.1.1	Sets of finite perimeter . . . . .	75
4.2	Weighted $BV$ functions . . . . .	76
4.3	A first characterization of $BV$ functions . . . . .	83
<b>5</b>	<b><math>BV</math> functions and parabolic problems: the second characterization</b>	<b>93</b>
5.1	The heat kernel in $\mathbf{R}^n$ . . . . .	94
5.2	Preliminary results for problems in a domain . . . . .	100
5.3	A second characterization of $BV$ functions . . . . .	104
<b>A</b>	<b>A brief introduction to interpolation theory</b>	<b>113</b>
A.1	Interpolation spaces . . . . .	113
A.1.1	Some interpolation estimates . . . . .	114
A.2	Real interpolation spaces . . . . .	115
A.2.1	Examples . . . . .	116
A.3	Complex interpolation spaces . . . . .	121
<b>B</b>	<b>Heat kernel estimates on domains</b>	<b>125</b>
<b>Appendix B</b>	<b>Heat kernel estimates on domains</b>	<b>125</b>
B.1	Gaussian upper bounds for heat kernels . . . . .	126
B.1.1	Some norm estimates . . . . .	126
B.2	Gaussian lower bounds . . . . .	127
B.2.1	The symmetric case . . . . .	128
B.2.2	The non-symmetric case . . . . .	130