(2.5) Theorem

Let M be a smooth compact orientable manifold of dimension even n and suppose that it has a distribution of oriented q-planes with q odd $(1 \leq q < n)$. *Then the Euler-Poincaré characteristic of M is null.*

3. Proof of theorems A and B.

(3.1) To every distribution there corresponds a smooth subbundle E_i of the tangent bundle with the fibre of dimension n_i. Denote the quotient bundle by $Q_i = TM/E_i$. From the Whitney duality formula $p(TM)=p(Q_i)p(E_i)$ it follows that

$$(3.2) \qquad P_r(TM) = P_r(Q_i)+P_{r-1}(Q_i)P_1(E_i)+\dots+p_1(Q_i)P_{r-1}(E_i)+p_r(E_i)$$

where the product between classes is the "cup product" in the ring $H^*(M; \mathbb{R})$.

If $2r > n_i$, then $r > n_i/2$ and hence $P_r(E_i) = 0$. If moreover

$$(3.3) \qquad P_h(Q_i)P_s(E_i) = 0 \qquad \forall h,s \geq 1,\ h+s = r$$

then

$$P_r(TM) = P_r(Q_i) \qquad 2r > n_i.$$

Notice that, since the Pontrjagin ring may have divisors of zero, condition (3.3) does not imply that either $P_h(Q_i) = 0$ or $p_s(E_i) = 0$. On account of our assumptions, from theorem (1.1) one can conclude that

$$P_r(Q_i) = 0 \qquad 2r > \max(n_1,\dots,n_k).$$

This proves theorem A.

(3.4) It is well known that if $E_i \subset TM$ is isomorphic to an integrable subbundle, then by Bott's theorem

$$P_r(Q_i) = 0 \qquad\qquad r > 2q_i$$

where $q_i = n-n_i$. Hence in the assumptions of theorem A, if $m = \max(n_1,\dots,n_k)<4q_i$, then one has for the integers h for which $m < h < 4q_i$

$$P_r(Q_i) = 0 \qquad\qquad 2r > h$$

whithout assuming that Q_i be integrable. This is meaningful if $k > 2$.

(3.5) Conversely let us assume Q_i to be integrable. Then, under our assumptions, one has at the same time

$$P_r(TM) = 0 \qquad P_r(Q_i) = o \qquad 2r > \max(m,4q_i)$$

hence an account of (3.2)

$$P_h(Q_i)\ P_s(E_i) = 0 \qquad\qquad \forall h,s \geq 1\ ,\ h+s = r.$$

This ends the proof of theorem B.

Remark

The results of theorem (2.1) hold in the more general situation of an almost multifoliated riemannian structures on a manifold, i.e.

$$TM = E_1 + \dots + E_k$$

where E_i are not necessarely complementary. Infact by increasing the number of distributions, with a suitable choice of the metric ([7]) it is possible go back to the previous situation.