SUMMARY. - A characterization of v-irreducible elements and of strongly v-irreducible elements of a distributive lattice $(L,\leq)$ was given by D.DRAKE and W.J. THRON in [1]. Among other things in [1] it was proven that an element $c \in L$ is v-irreducible iff one can identify $(L,\leq)$, by means of a lattice isomorphism f, with a sublattice $(L',\leq)$ of the power set $P(X)$ of a suitable set X, in such a way that $f(c)$ is the closure in L' of an element $x \in X$ (i.e. $F(c)$ is the minimum element in L', with respect to the set inclusion, including x).

As is well-known an element of a distributive lattice is v-irreducible iff it is v-prime. This property is exploited is an essential manner in [1]. Now then in our paper we took this property as a starting point for a characterization of v-prime and of strongly v-prime elements of any partially ordered set (in particular of any lattice). Here, on the analogy of some characterization of v-prime elements and of strongly v-prime elements of a lattice, an element c of a partially ordered set (shortly "poset" $(S,\leq)$ is said v-prime iff the subset $D_c = \{s \in S : c \nleq s\}$ is v-directed, i.e. $D_c = \emptyset$ or for every $x_1, x_2 \in D_c$ (for every $x_1,\ldots,x_n \in D_c$) there exists $t \in D_c$ such that $x_1 \leq t$ and $x_2 \leq t$ ($x_i \leq t$ for every $i = 1,\ldots n$); moreover c is said strongly v-prime if $D_c = \emptyset$ or D_c has maximum element. Then we prove than an element $c \in S$ is v-prime in $(S,\leq)$ iff we can identify $(S,\leq)$ by means of an order isomorphism f, with a set (but not necessarily a lattice) of sets of the type of [1] in such a way that $f(c)$ is the closure in $(f(S),\leq)$ of an element of $\cup f(S)$; moreover we prove that c is strongly v-prime in $(S,\leq)$ iff for all function f of the above type the set $f(c)$ is the closure in $(f(S),\leq)$ of an element of $\cup f(S)$.

N. 1 PRELIMINARY CONSIDERATIONS.

We recal that a lattice is said a set lattice (see [1] p. 57) iff its elements are subsets of a suitable set X and the order relation is the set inclusion; in particular if the lattice is a sublattice of the power set $\mathcal{P}(X)$ then it is called a proper set lattice.

More generally we shall say that a set lattice $(L',\subseteq)$ is a "U-proper set lattice" iff the lattice join is equal to the set union.

We recall also that a proper set representation of a lattice $(L,\leq)$

is an ordered pair $((L',\subseteq),f)$, where $(L',\subseteq)$ is a proper set lattice and f is an isomorphism from $(L,\leq)$ onto $(L',\subseteq)$. If $(L',\subseteq)$ is a $\cup$-proper set we shall call $((L',\subseteq),f)$ a "$\cup$-proper set representation". We want to extend the previous definitions to the case of an arbitrary partially ordered set.

In the meantime we observe that the lattice join is equal to the set union in a set lattice $(L',\subseteq)$ iff the following property holds:

(i) For every $A_1,\dots,A_n \in L'$ $\bigcup_{i=1}^{n} A_i$ is equal to the set intersection of all the elements of L' which include every $A_1,\dots,A_n$.

As a consequence of this fact we shall say that a poset is a $\cup$-proper set poset iff its elements are subsets of a suitable set X, the order relation is the set inclusion and property i) holds(1); thus we shall say that the ordered pair $((S',\subseteq),f)$, where $(S',\subseteq)$ is a $\cup$-proper set poset and f is a function, is a $\cup$-proper set representation of a poset $(S,\leq)$ iff f is an order isomorphism from S onto S' (i.e. f is a bijective isotone function from S onto S' and f^{-1} is also isotone).

In the following we shall prove the next properties:

1) An element c of a poset $(S,\leq)$ is v-prime iff a $\cup$-proper set representation $((f(S),\subseteq),f)$ of $(S,\leq)$, exists such that $f(c)$ is a point closure $f(S)$. Moreover if $(S,\leq)$ has at least a v-prime element then a $\cup$-proper set representation $((f(S),\subseteq),f)$ of $(S,\leq)$ exists such that f maps every v-prime element of $(S,\leq)$ in a point closure in $f(S)$.

2) An element c of the poset $(S,\leq)$ is strongly v-prime iff for every

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(1) In this case if a common upper bound of $A_1,\dots,A_n$ does not exist in L' then we put the above mentioned set intersection equal to $\bigcup_{Y\in L'} Y$.

$\mathcal{U}$-proper set representation $((f(S), \subseteq), f)$ of $(S, \leq)$[2] $f(c)$ is a point closure.

N. 2. A BRIEF REVIEW OF PREORDERED SETS.

Let S be a set and $\lesssim$ a preorder relation for S(i.e. $\lesssim$ exhibits the transitive and reflexive properties). All the most important notions about a poset can be extended to a preordered set (e.g. upper bound, lower bound, maximum, minimum, ℓ.u.b., g.ℓ.b., etc.); thus a right tail of a preordered set $(S,\lesssim)$ will be every $Y \subseteq S$ such that $\forall\, x,y \in S: x \in Y$ and $x \lesssim y \Longrightarrow y \in Y$.

We observe that if $Y_1 \subseteq S$ then the set $r(Y_1) = \{x \in S : x$ is an upper bound of $Y_1\}$ is a rigth tail of $(S,\lesssim)$; in particular the principal filter $r(y) = r(\{y\})$ generated by $y \in S$ is a right tail of $(S,\lesssim)$. Moreover

(ii) $r(X) = \bigcap_{x \in X} r(x)$ and if X is a right tail then $X = \bigcup_{x \in X} r(x)$.

Now let $\mathcal{C}$ be a subset of $\mathcal{P}(S)$ (the power set of S), x an element of S and $\mathcal{C}_x = \{X \in \mathcal{C} : x \in X\}$. Then we define, for every $x,y \in S$

(j) $$x \lesssim y(\mathcal{C}) \text{ iff } \mathcal{C}_x \subseteq \mathcal{C}_y .$$

Clearly the defined relation is a preorder relation. Moreover if $\mathcal{C}'$ is the set of set complements of the elements of $\mathcal{C}$ it follows, since $\mathcal{C}_x \subseteq \mathcal{C}_y$ iff $\mathcal{C}'_y \subseteq \mathcal{C}'_x$,

(jj) $$x \lesssim y(\mathcal{C}) \text{ iff } y \lesssim x(\mathcal{C}') .$$

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(2) We shall prove that there exists at least a $\mathcal{U}$-proper set representation of $(S,\leq)$.