1 Introduction

In the last decades, economists have been increasingly concerned with the issue of growing personal income inequality. Strong emphasis has been laid on measuring the latter, assessing its causes, and discussing the implied redistributive policies.[1] As to the economy-wide implications of inequality, income distribution has been shown to affect growth performance, through such channels as political and institutional mechanisms (e.g., Persson and Tabellini, 1994; Benabou, 1996), capital market imperfections (e.g., Piketty, 1997; Aghion *et al.*, 1999), or the structure of aggregate demand (e.g., Echevarria, 2000; Zweimuller, 2000).

This paper focuses on the relationship between personal income distribution and the behaviour of micro and macro markets, within a different perspective – namely, that of market competitiveness as measured by the degree of monopoly power. In particular, we study how distributive shocks on the degree of income dispersion affect the equilibrium of a monopolistic competitive market *à la* Dixit and Stiglitz (1977). We can think of two reasons why modeling income distribution shocks within this framework may prove useful. In a micro perspective, it allows to establish a well defined connection between income dispersion, firms' profitability and product differentiation, as measured by the equilibrium number of varieties. On the other hand, the popularity of the the Dixit-Stiglitz model in the macroeconomics of imperfect competition may suggest interpreting our results in terms of a link between personal distribution of income, aggregate demand, and the cyclical behaviour of aggregate mark-up.

Our discussion is organized as follows. In Section 2 we re-cast the Dixit-Stiglitz approach to product differentiation into a non-homothetic structure of preferences, which allows introducing income heterogeneity in a meaningful way; we then build the demand side of the model, by parametrizing income dispersion through a mean preserving spread. In Section 3 we consider the effects of changes in income dispersion on the short and long run equilibria of the model, under the standard negligibility assumption that each firm neglects the external effects of its own price decision on the aggregate price – income dispersion turns out to influence only the degree of product differentiation. In Section 4 we show that removing the negligibility assump-

[1] Recent comprehensive discussions of these issues are provided by Champernowne and Cowell (1998) and Lambert (2001).

tion results in income dispersion affecting also the firms' price and quantity choices, through changes in the equilibrium mark-up. Concluding remarks are gathered in Section 5.

2 Market demand and income dispersion

We consider a population of consumers who differ only in their income I. The latter is distributed according to a continuous, differentiable, unimodal density $f(I,\theta)$, defined over the positive interval $[I_{\min}, I_{\max}]$. In order to focus on the effects of income inequality, in the sequel we interpret the parameter $\theta \in \Theta$ as a mean preserving spread, so that an increase in θ can be seen as an increase in income dispersion which leaves average income unchanged.

Consumers' preferences are identical and represented by the following utility function:

$$U = U\left(x_0, \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right) \tag{1}$$

where x_0 is a numéraire homogeneous commodity and x_i , $i = 1, ..., n$, are the different varieties of a CES composite differentiated good $y = \left(\sum_{i=1}^{n} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$, where $\sigma > 1$ is the constant elasticity of substitution across varieties. We depart from the standard specifications of this Dixit-Stiglitz framework, by assuming that (1) is non-homothetic, in order to generate Engel's curves which are not unit-elastic in income. Clearly, the strict proportionality between demand and income associated to homothetic preferences would not leave any role to income distribution in the analysis of demand, the only relevant parameter being the income mean (aggregate) value.

Each consumer maximizes (1), given the linear budget constraint

$$x_0 + \sum_{i=1}^{n} p_i x_i = I$$

Through a two-stage budgeting procedure, the solution of this maximization problem yields the following demand function for each variety x_i:

$$x_i^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{s}{q} \tag{2}$$

where s is the consumer's expenditure in the differentiated good and q is the (dual) price index defined as

$$q = \left(\sum_{i=1}^{n} p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

By substituting (2) into (1) for all x_i and recalling that $x_0 = I - s$, it is possible to determine the optimal value of s as a function of I and q, $s = s(I, q)$, and therefore the marshallian demand for the differentiated good and the numeraire.

$$\begin{aligned} y &= \frac{s(I, q)}{q} \\ x_0 &= I - s(I, q) \end{aligned}$$

The marshallian demand for variety x_i is therefore

$$x_i^d = \left(\frac{p_i}{q} \right)^{-\sigma} \frac{s(I, q)}{q}$$

By aggregating over consumers we obtain the market demand for variety i:

$$X_i^d = \left(\frac{p_i}{q} \right)^{-\sigma} \frac{1}{q} S(q, \theta) \tag{3}$$

where

$$S(q, \theta) = \int_{I_{\min}}^{I_{\max}} s(I, q) f(I, \theta) \, dI$$

Given the heterogeneity of consumers with respect to income, market demand is in principle affected by the parameters of income distribution. However, with homethetic preferences the s function (and the demand function) would be linear in income and the mean preserving spread parameter would not affect aggregate expenditure S. Our non-homotheticity hypothesis allows for a concave or convex shape of s, so that θ actually influences S and market demand X_i^d. In particular, the following proposition holds:

Proposition 1 *If the differentiated good is a necessary good,* $\partial X_i^d/\partial\theta < 0$*, i.e. an increase in income dispersion decreases market demand. If the differentiated good is a luxury good,* $\partial X_i^d/\partial\theta > 0$*, i.e. an increase in income dispersion raises market demand.*

The proof is omitted, as it is a direct application of the general result that the expected value of a concave (convex) function is decreasing (increasing) in any mean preserving spread parameter (Hirshleifer and Riley, 1992, p. 112).

This result is rather intuitive. An increase in income dispersion implies an increase in the density of low income and high income consumers, with a shrinking of the middle class. The Engel curve of a necessary good is concave and therefore the increase in demand from the newly rich consumers does not compensate the decrease in demand by the newly poor consumers. The opposite applies in the case of a luxury good. A simple example where aggregate expenditure is a linear function of the dispersion parameter is provided below.

Example. Consider the non-homothetic utility functions (Chou and Talmain, 1996)

$$
\begin{aligned}
U_1 &= \sqrt{x_0} + \ln\left(\left(\sum x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right) \\
U_2 &= -\frac{1}{x_0} + \ln\left(\left(\sum x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right)
\end{aligned}
$$

to which there correspond the individual marshallian demand functions for variety i:

$$
\begin{aligned}
x_{i1}^d &= \left(\frac{p_i}{q}\right)^{-\sigma}\frac{2}{q}\left(\sqrt{1+I}-1\right) \\
x_{i2}^d &= \left(\frac{p_i}{q}\right)^{-\sigma}\frac{1}{q}\left(I+\frac{1}{2}-\frac{1}{2}\sqrt{(4I+1)}\right)
\end{aligned}
$$

where x_{i1}^d is concave and x_{i2}^d is convex in income. Assume now that income is distributed according to the density

$$f(I,\theta) = \theta + 6(1-\theta)I(1-I)$$

defined over the support $[0,1]$.[2] By aggregating the individual demand curves, we obtain market demand functions linear in θ of the type:

$$X_{i1}^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{1}{q} (\alpha_1 - \beta_1 \theta)$$
$$X_{i2}^d = \left(\frac{p_i}{q}\right)^{-\sigma} \frac{1}{q} (\alpha_2 + \beta_2 \theta)$$

where α_j and β_j $(j = 1, 2)$ are positive numbers.[3] As expected, X_{i1}^d is decreasing (X_{i2}^d is increasing) in θ.

We now apply the above demand framework in the analysis of market equilibrium.

3 Pricing and market equilibrium: the case with exogenous mark-up

Following the standard Dixit-Stiglitz approach, we assume that each firm faces the following cost function

$$C(x_i) = a + cx_i$$

If each firm maximizes its own profits under the demand constraint (3) and taking q as given, the symmetric short run equilibrium price is

$$p_i = p = c\frac{\sigma}{\sigma - 1} \tag{4}$$

and since $q = pn^{\frac{1}{1-\sigma}}$,

$$X_i = X = \frac{S\left(pn^{\frac{1}{1-\sigma}}, \theta\right)}{pn} \tag{5}$$

According to equation (4), the equilibrium mark-up is fully determinated by the exogenous cost and demand parameters, and is therefore independent

[2] This density, a mixture of a uniform and a quadratic beta distribution, is unimodal and symmetric. It is easy to check that the parameter $\theta \in [0,1]$ is a mean preserving spread, so that an increase in θ increases income dispersion.

[3] In particular we have $\alpha_1 \simeq 0.4426$, $\beta_1 \simeq 0.0047$, $\alpha_2 \simeq 0.1443$, $\beta_2 \simeq 0.0074$.

of the properties of income distribution. Indeed, this result is due to two main assumptions, both implicit in the Dixit-Stiglitz approach. The first is the CES formulation for the composite differentiated good; the second is the so-called negligibility hypothesis, according to which each firm is assumed to neglect the effect of its own price decisions on the aggregate price q. We shall relax this latter assumption in the next section.

Equation (5) shows that the distribution parameter affects the short run optimal production decision: at the given price the changes in the demand size induced by distributional shocks are met through quantity adjustments.

Let us now consider the long run market equilibrium. By using (4) and (5) in the zero profit condition $(p-c)\,X=a$, we get

$$\frac{S\left(pn^{\frac{1}{1-\sigma}},\theta\right)}{pn}=\frac{a}{c\left(\dfrac{1}{\sigma-1}\right)} \tag{6}$$

which makes it clear that the distributional parameter θ affects, through the above size-effect on demand, the equilibrium number of firms and therefore the degree of product differentiation. The role of θ is synthetized in the following proposition.

Proposition 2 *If the intersectoral elasticity of demand for the composite differentiated good y, $\eta_{yq}=-\left(q/y\right)\left(\partial y/\partial q\right)$, is lower than the intrasectoral elasticity σ, then an increase in income dispersion reduces (increases) the equilibrium number of varieties of a necessary (luxury) differentiated good.*

Proof. By implicit differentiation of (6), we get

$$\frac{dn}{d\theta}=-\frac{\frac{\partial S}{\partial\theta}}{\frac{S}{pn^{2}}\left(\frac{q}{S}\frac{\partial S}{\partial q}\frac{n}{q}\frac{\partial q}{\partial n}-1\right)}$$

Given that $\left(q/S\right)\left(\partial S/\partial q\right)=1-\eta_{yq}$, and that $\left(n/q\right)\left(\partial q/\partial n\right)=1/\left(1-\sigma\right)$, the denominator is negative if $\eta_{yq}<\sigma$. This condition ensures uniqueness of equilibrium (Dixit and Stiglitz, 1977,p.300),[4] and if it is verified, $sign\left[dn/d\theta\right]=sign\left[\partial S/\partial\theta\right]$. Making use of Proposition 1, Proposition 2 then follows straightforwardly.□

[4] In a Chamberlinian framework, it amounts to imposing that the *dd* curve be more elastic than the *DD* curve.

The simple analysis of this section shows clearly that in the standard Dixit-Stiglitz framework of product differentiation, income dispersion plays indeed a role in the long run configuration of market structure, in that it affects the number of available varieties; but it seems to be irrelevant as far as the competitiveness of market is concerned: the price over cost ratio and each firm long run equilibrium output are independent of the distributive parameter.

It may be noticed that, in a love-for-variety framework, this effect on the degree of product differentiation is a non negligible, side welfare effect of distributional shocks, which affects all consumers utility in the same direction, independently of their displacement in the scale of incomes.

4 Pricing and market equilibrium: the case with endogenous mark-up

In the simple model of the previous section income dispersion exerts only a size effect on demand, the direction of which depends on the concavity or convexity of the Engel's curve. However, there exists in principle an additional transmission mechanism of distributive shocks to the goods market, namely that of market demand elasticity. This issue has already been dealt with in a homogenous product setup: in that framework an increase in income dispersion reduces for a relevant price range both the size and the elasticity of market demand (e.g., Benassi, Chirco and Scrimitore, 2002).

Within the love-for-variety models of product differentiation, the CES formulation of preferences is consistent with an endogenous mark-up only if the above mentioned negligibility hypothesis is abandoned and the so-called price index effect is taken into account. The idea that firms do not neglect the change in the price index q induced by their own pricing decisions has been introduced by Yang and Heijdra (1993) and generates an endogenous mark-up behaviour in the macroeconomic analyses by Bratsiotis and Martin (1999), Wu and Zhang (2000), Linnemann (2001), Benassi, Chirco and Colombo (2002).

If the perceived elasticity of the price index with respect to the individual price is not nil, $\nu_{qp_i} = (p_i/q)\ \partial q/\partial p_i \neq 0$, then the own price elasticity (in absolute value) of market demand (3) becomes

$$\varepsilon_{ii} = \sigma - (\sigma - 1)\,\nu_{qp_i} + \frac{\partial S}{\partial q}\frac{q}{S}\nu_{qp_i} \tag{7}$$

Evaluated at the symmetric equilibrium, we can write (7) as

$$\varepsilon_{ii} = \sigma - \frac{1}{n}(\sigma - 1) + \frac{1}{n}\frac{\partial S}{\partial q}\frac{q}{S} \tag{8}$$

If we assume that the demand elasticity of the composite differentiated good with respect to q be unity, so that the elasticity of expenditure with respect to price is equal to zero,[5] equation (8) collapses to

$$\varepsilon(\sigma, n) = \sigma - \frac{1}{n}(\sigma - 1)$$

The short run profit maximization then yields the following symmetric equilibrium solution for price and production:

$$p_i = p = c\left(\frac{\varepsilon(\sigma, n)}{\varepsilon(\sigma, n) - 1}\right) \tag{9}$$

$$X_i = X = \frac{S(\theta)}{pn} \tag{10}$$

Equations (9), (10) and the zero profit condition determine the long run equilibrium price, quantity and number of firms:

$$\begin{aligned} p &= c\left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{S(\theta)}{S(\theta) - a}\right) \\ X &= \left(\frac{a(\sigma - 1)}{c}\right)\left(\frac{S(\theta) - a}{S(\theta) + a(\sigma - 1)}\right) \\ n &= \frac{S(\theta)}{a\sigma} + \frac{\sigma - 1}{\sigma} \end{aligned}$$

[5] As is well known, this condition is satisfied when the U function is of the Cobb-Douglas type - as in most common macroeconomic applications of this framework. Indeed, it allows closed form solutions in the presence of a price-index effect (Yang and Heijdra, 1993, p.297). It is also satisfied by the utility functions used in the example of Section 2, which (like the Cobb-Douglas) yield the unit price-elasticity property, but (unlike the Cobb-Douglas) exhibit non-homotheticity.

Now, the distributive parameter θ influences not only the optimal number of varieties, but also the price over cost ratio and the individual level of production in the long run equilibrium. In particular, the following proposition holds:

Proposition 3 *If the differentiated good is a necessary (luxury) good, in the long run equilibrium an increase in income dispersion increases (decreases) the equilibrium price, while it decreases (increases) the number of varieties and the level of production of any firm.*

Proof. By noticing that

$$\begin{aligned}
\frac{\partial n}{\partial \theta} &= \frac{1}{a\sigma}\frac{\partial S}{\partial \theta} \\
\frac{\partial p}{\partial \theta} &= -c\left(\frac{\sigma}{\sigma-1}\right)\frac{a}{\left(S\left(\theta\right)-a\right)^2}\frac{\partial S}{\partial \theta} \\
\frac{\partial X}{\partial \theta} &= \frac{a^2\sigma\left(\sigma-1\right)}{c\left(S\left(\theta\right)+a\left(\sigma-1\right)\right)^2}\frac{\partial S}{\partial \theta}
\end{aligned}$$

and making use of Proposition 1, Proposition 3 follows immediately.□

Therefore, on the basis of this simple framework, we expect that in the market for a differentiated non luxury good a stronger concentration of incomes around the mean value (and therefore a lower degree of inequality) increases demand, the level of production and the spectrum of available varieties; still more interestingly, this is accompanied by an increase in demand elasticity and a reduction in the equilibrium mark-up. In these markets, concentration of personal incomes implies a reduction of the profit margins. The opposite applies in markets for luxuries, where a greater homogeneity in incomes implies a decrease in demand and production and an increase in profit margins.

The economic intuition behind these results is not difficult to capture. A lower degree of income dispersion and greater income concentration imply a growing share of demand coming from middle income consumers. For non-luxury differentiated goods, this change in the income-class composition of demand results into an increase in market demand, while the demand for luxuries shrinks. For a given number of firms, these demand shocks are accompanied by a positive comovement in the firms' production and profit levels, which fosters an entry-exit mechanism. The latter changes the competitive structure of the market. In the above setup, distributional shocks of the mean preserving spread type can therefore be seen as generating demand shocks with a procyclical price-elasticity pattern.

5 Conclusions

Consumers' heterogeneity is often referred to as an important element in explaining basic properties of market behaviour. When thinking about income, which is definitely one of the main dimensions of heterogeneity, a natural question in this respect is whether a more egualitarian distribution contributes to creating a more competitive market environment, through mechanisms based on market demand elasticity. This issue has important implications in both an industrial organisation, and a macroeconomic perspective. It identifies a well defined demand component in the endogeneization of market structure, and it suggets a potential link between the personal distribution of income, the competitive pattern of the economy, and the functional distribution of income.

Clearly, the answer to this question depends on the key features of the non-competitive structure under investigation – the relevant distinctions being, e.g , homogeneous *vs* differentiated product, durable *vs* non durable goods, price *vs* quantity competition, etc.

In this paper we have studied this problem within the Dixit-Stiglitz model of product differentiation, which has been widely used both in the industrial economics and in the macroeconomic literature. Our main result is that any tendency towards income concentration brings about a shift in market demand, which is associated to a procyclical change in demand elasticity when the latter is endogeneized through the so-called price index effect. However, the direction of these co-movements in demand and demand elasticity depends on the shape of the Engel's curve: while for necessary goods income concentration reduces the firms' market power, for luxuries it lowers demand and deepens the firms' profit margins.

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