When evaluated at these prices, and at $a' = -\frac{4}{15} + \epsilon$, and $b = \frac{2}{3}$, the profits of firm 1 turn out to be

$$\frac{5}{6}\left(\frac{268}{675} - \frac{4}{15}\epsilon - \frac{1}{3}\left(-\frac{4}{15} + \epsilon\right)^2\right)\left(\frac{\frac{44}{135} - \frac{2}{3}\epsilon + \frac{2}{3}\left(-\frac{4}{15} + \epsilon\right)^2}{\frac{14}{15} - \epsilon} + \epsilon\right) > \frac{28}{225}$$

for arbitrarily small positive values of ϵ. This is enough to prove that, for $w = \frac{1}{5}$, the solutions (17)-(20) are not subgame perfect equilibria and allows us to establish that for $w = 1/5$ there exists only a subgame perfect symmetric Nash equilibrium in prices and locations, defined by equations (6)-(9).

4 Remarks and conclusions

In this paper we have analysed the effects of the consumers' concentration towards the middle of the space of product characteristics, in a a model of horizontal differentiation with quadratic transportation costs. The consumers' density is assumed to be symmetric and trapezoidal; if the size of the market is normalized to 1, this allows to consider the lenght of the shortest base as a mean preserving spread of consumers' preferences. Clearly, the traditional uniform distribution and a symmetric triangular distribution can be nested into this setup as limit cases.

We have proved that as far as the shortest base is positive - i.e. the distribution is differentiable at 1/2 - a symmetric subgame perfect Nash equilibrium exists in the two stage price-location game. The result we achieve is rather intuitive: starting from the optimal solution obtained under the standard uniform distribution, as preferences become more concentrated around the middle, both firms move inwards and reduce the degree of product differentiation. This clearly reinforces price competition and results in lower equilibrium prices. This result is consistent with a more general intuition that homogeneity of consumers might have important implications in terms of reducing the firms'market power (Benassi, Chirco, and Scrimitore, 2002).

Moreover, our discussion shows that the asymmetric equilibria identified by Tabuchi and Thisse may coexist with the above symmetric equilibrium. For a relevant range of values of our mean preserving spread parameter - when preferences become sufficiently concentrated - two asymmetric subgame perfect equilibria appear, with one firm producing a relatively 'average' product, and the other firm choosing to locate outside the characteristics space. Once one firm decides to produce a product which meets the taste of the large share of consumers located around the middle, the other firm finds it optimal to avoid a destructive price competition by choosing a product with 'extreme' and 'out of market' characteristics. However, this peculiar location choice requires that a low price is charged, in order to capture at least the consumers located at the nearest tail of the distribution. This solution is such that as w increases within its admissible range - the distribution becomes more dispersed - both firms locate inwards and decrease their price. As the relative weight of the tails increases, the firm producing outside the market area perceives an incentive to make its product more attractive for the growing share of consumers it may patronize - those located at its nearest tail. The firm producing inside the market area, perceiving no competition at the other tail, challenges its rival by locating further towards the middle. These movements result in a tougher price competition.

While the simple setup discussed in this paper allows for an explicit general solution which covers the situations previously discussed in the literature, it is nevertheless clear that the relation between any concentration index of the consumers' preferences and the properties of equilibria should be framed in a more general setting, independently of the possibility of defining analytical solutions. This is an important issue of the research agenda on product differentiation.