Katsoulacos 1991). The problem of the optimal prices and locations has been explicitly solved by Tabuchi and Thisse (1995) with a triangular and symmetric distribution. They show that, given that distribution, any symmetric location around the middle cannot be an equilibrium. Indeed, two asymmetric equilibria arise, characterized by strong product differentiation between the firms, with one of them locating outside the support of the customer distribution.Their results, however, heavily depend on the non differentiability of the consumers density function, which generates a discontinuity of the reaction functions in correspondence of any symmetric location.

In this paper, we aim at extending Tabuchi and Thisse analysis in two directions. We offer a simple parametrization of the degree of consumers' concentration around the middle - which include the uniform and the triangular distribution as limit cases. This allows us to solve the price-location problem as a function of the degree of consumers concentration. Within this setup, we are able to show that a symmetric equilibrium exists, provided the density is differentiable at the center of its support. Moreover, we are able to give some theoretical support to the idea that a higher concentration of consumers around the center induces firms to reduce the optimal product differentiation. Finally, we find that the asymmetric equilibria identified by Tabuchi and Thisse may arise for a lower degree of consumers concentration than that implied by the triangular distribution and that these asymmetric equilibria may coexist with a symmetric one.

The paper is organized as follows. In section 2 we describe the basic model and discuss the simple parametrization of consumers' concentration adopted in the sequel. The explicit solution of the price-location problem is presented in section 3. Some comments and concluding remarks are provided in Section 4.

2 The model

Let us consider a market for a horizontally differentiated product, where the population of consumers is normalized to 1. Consumers, indexed with x, are distributed over the interval $[0,1]$, according to a density $f(x,w)$, where the parameter w that can be viewed as a concentration index of the consumers' tastes. More precisely, the density $f(x,w)$ is characterized as follows:

$$\begin{array}{lll} & f(x,1)=1, & \text{for } x\in[0,1] \\ & f(x,0)=2-2\left|2x-1\right|, & \text{for } x\in[0,1] \\ \text{for } 0<w<1 & f(x,w)=\dfrac{4}{1-w^2}x & \text{for } x<\dfrac{1-w}{2}, \\ & f(x,w)=\dfrac{2}{1+w} & \text{for } x\in\left[\dfrac{1-w}{2},\dfrac{1+w}{2}\right], \\ & f(x,w)=\dfrac{4}{1-w^2}(1-x) & \text{for } x<\dfrac{1+w}{2} \end{array}$$

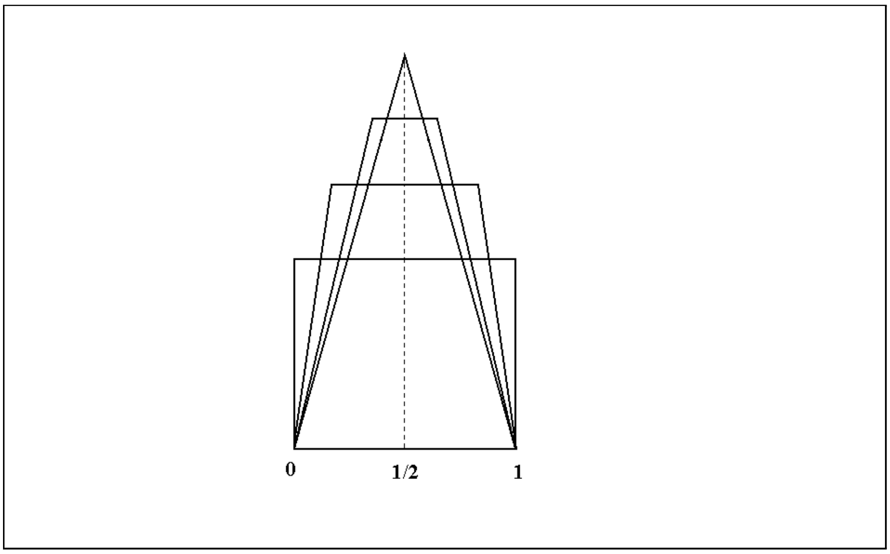

Figure 1: The density function for different values of the concentration parameter

As shown in Figure 1, $f(x,w)$ is symmetric around $x = 1/2$; for $w = 1$ it describes a uniform distribution while, as w decreases it concentrates towards the middle becoming trapezoidal and collapsing to a triangle for $w = 0$. Roughly speaking, our density is a trapezoid, with longest base equal to 1, shortest base equal to w and altitudo equal to $\frac{2}{1+w}$. For a given $w \in (0,1]$, we define 'central interval' the interval $x \in \left[\frac{1-w}{2}, \frac{1+w}{2}\right]$, while we call 'left external' and 'right external' interval respectively the intervals $x \in \left[0, \frac{1-w}{2}\right)$ and $x \in .\left(\frac{1+w}{2}, 1\right]$.

In this framework we consider a duopoly model in which both firms, firm 1 and firm 2, produce a differentiated product at a constant and equal to zero marginal cost. The location x chosen by each firm represents the good it decides to produce: the ideal consumer's product may match with the product offered, otherwise consumers choose to buy a "less than ideal" product paying a transportation cost that we consider quadratic in distance. Each consumer takes at most one unit of the product, so that total demand for the good offered by the firm located in x is given by the number of customers it patronizes. In the sequel we shall assume full market coverage.

Let us denote with a the distance of firm 1 from the origin, while b is the distance of firm 2. In order to exclude the possibility of leapfrogging by either firms we assume $a < b$ - where $a \in (-\infty, \infty)$ and $b \in (-\infty, \infty)$ - and marginal consumer lying between the two firms. As is well known, the price-location problem is a two-stage game in which at the first stage the firms choose their location and at the second stage choose their prices. The game is simultaneous.

The optimal firms' behaviour obviously differs according to the value of w. The results in terms of optimal locations are well known in the literature when $w = 1$ and when $w = 0$: in the unconstrained Hotelling game with a uniform distribution of consumers the firms maximize profits by locating at $-1/4$ e $5/4$ (Lambertini, 1994); moreover, Tabuchi and Thisse (1995) demonstrate that with a triangular distribution two asymmetric equilibria arise, $\left(-\sqrt{6}/9, 5\sqrt{6}/18\right)$ and $\left(1 - 5\sqrt{6}/18, 1 + \sqrt{6}/9\right)$. The following analysis will focus on the price-location equilibria for intermediate values of the parameter w, i.e. when the density becomes trapezoidal.

3 Consumer concentration and equilibrium prices and locations

We look for a subgame perfect equilibrium through backward induction, solving first for the prices and then for the locations as a function of the exogenous parameter w and the optimal prices determined in the first stage. Notice that if firm 1 and 2 set a price respectively equal to p_1 and p_2 being located respectively in a and b, the above hypotheses on transportation costs, unit demand and full market coverage imply that the marginal consumer's location is

$$z = \left(\frac{1}{2}\left[\frac{p_2 - p_1}{b - a} + b - a\right] + a\right) \tag{1}$$

Clearly, given the shape of our density, the firms' reaction functions in both stages of the game will be different according to the fact that the firms know that their behaviour implies that the marginal consumer lies in the 'central interval' or in the two external intervals, i.e. $z \in \left[\frac{1-w}{2}, \frac{1+w}{2}\right]$ - central interval - or $z \in \left[0, \frac{1-w}{2}\right)$ and $z \in .\left(\frac{1+w}{2}, 1\right]$ - external interval. We solve the model under both conjectures and verify under which conditions one or more equilibria exist in which conjectures are fulfilled. Notice that, given the simmetry of the density, the possible existence of a subgame perfect equilibrium such that $z \in \left[0, \frac{1-w}{2}\right)$ implies the existence of a specular equilibrium, with the marginal consumer lying in a specular position within the interval $\left(\frac{1+w}{2}, 1\right]$. This allows to restrict the analysis to one external area only.

3.1 The marginal consumer lies in the central interval

Given the hypothesis of unit consumers' demand and given our normalization, the market demand for each good corresponds to its market share. Therefore,