List of symbols

Let $1 \leq k \leq \infty$, $N \in \mathbb{N}$, $0 < \alpha < 1$, $T > 0$, $a < b$, u real valued function.

$\mathbb{R}^N$	euclidean N-dimensional space
$Q(a,b)$	$\mathbb{R}^N \times (a,b)$
Q_T	$Q(0,T)$
(X,d)	a metric space X endowed with the distance d
$(\cdot\|\cdot)$	scalar product or, in general, duality
$\|x\|$	euclidean norm of $x \in \mathbb{R}^N$
$B_\rho(x)$	open ball for the euclidean distance with centre x and radius ρ
$\|E\|$	Lebesgue measure of a given set E
χ_E	characteristic function of e set E
$\mathrm{supp}\,\mathrm{u}$	support of a given function u
$D_i u$	partial derivative with respect to x_i
$\partial_t u$	partial derivative with respect to t
$D_{ij} u$	$D_i D_j u$
Du	$(D_1 u, ..., D_N u)$
$D^2 u$	hessian matrix $(D_{ij} u)_{i,j=1,\dots,N}$
$\|Du\|^2$	$\sum_{j=1}^N \|D_i u\|^2$
$\|D^2 u\|^2$	$\sum_{i,j=1}^N \|D_{ij} u\|^2$
$f^+,\ f^-$	positive part $f \vee 0$ and negative part $-(f \wedge 0)$ of f
$\mathbf{1}$	function identically equal to 1 everywhere
$\mathcal{L}(X)$	space of bounded linear operators from X to X
$C_b(\mathbb{R}^N)$	space of bounded continuous functions in $\mathbb{R}^N$
$C_b^j(\mathbb{R}^N)$	space of real functions with derivatives up to the order j in $C_b(\mathbb{R}^N)$
$C^\alpha(\mathbb{R}^N)$	space of Hölder continuous functions
$C^\alpha_{loc}(\mathbb{R}^N)$	space of Hölder continuous functions in Ω for all bounded open set $\Omega \subset \mathbb{R}^N$
$C^{k+\alpha}(\mathbb{R}^N)$	space of functions such that the derivatives of order k are α-Hölder continuous
$C_c^\infty(\mathbb{R}^N)$	space of test functions
$L^p(\mathbb{R}^N)$	usual Lebesgue space
$L_c^\infty(\mathbb{R}^N)$	space of all bounded measurable functions on $\mathbb{R}^N$ having compact support
$\mathcal{S}(\mathbb{R}^N)$	Schwartz space
$\mathcal{S}'(\mathbb{R}^N)$	space of tempered distributions
$B_b(\mathbb{R}^N)$	space of bounded Borel functions

$C_0(R^N)$	space of continuous functions tending to 0 for $\lvert x\rvert$ tending to $+\infty$
$C_0(B_\rho)$	space of continuous functions in B_ρ vanishing on the boundary
$BUC(Q(a.b))$	space of bounded and uniformly continuous functions in $Q(a.b)$
$C^{2,1}(Q(a,b))$	space of functions continuous with their indicated derivatives
$C_b^{2,1}(Q(a,b))$	space of functions having bounded time derivative and bounded space derivatives up to the second order
$BUC^{2,1}(Q(a,b))$	subspace of $C_b^{2,1}(Q(a,b))$ consisting of all functions for which u_t and $D_x^\alpha u$, $\lvert\alpha\rvert = 2$ are uniformly continuous in $Q(a,b)$
$C^{2+\alpha,1+\frac{\alpha}{2}}(Q(a,b))$	space of functions such that $\partial_t u$ and $D_{ij}u$ are α Hölder continuous with respect to the parabolic distance
$W_k^j(\mathbb{R}^N)$	space of functions $u \in L^k(\mathbb{R}^N)$ having weak space derivatives up to the order j in $L^k(\mathbb{R}^N)$
$W_k^{2,1}(Q(a,b))$	space of functions $u \in L^k(Q(a,b))$ having weak space derivatives $D^\alpha u \in L^k(Q(a,b))$ for $\lvert\alpha\rvert \le 2$ and weak time derivative $\partial_t u \in L^k(Q(a,b))$
$\lVert u\rVert_{W_k^{2,1}(Q(a,b))}$	$\lVert u\rVert_{L^k(Q(a,b))} + \lVert\partial_t u\rVert_{L^k(Q(a,b))} + \sum_{1\le\lvert\alpha\rvert\le 2} \lVert D^\alpha u\rVert_{L^k(Q(a,b))}$
$[u]_{\alpha,\frac{\alpha}{2};Q_T}$	$\sup_{(x,y)\in\mathbb{R}^N, t\in(0,T)} \frac{\lvert u(x,t)-u(y,t)\rvert}{\lvert x-y\rvert^\alpha} + \sup_{s\neq t, x\in\mathbb{R}^N} \frac{\lvert u(x,t)-u(s,x)\rvert}{\lvert t-s\rvert^{\frac{\alpha}{2}}}$
$\lvert u\rvert_{\alpha,\frac{\alpha}{2};Q_T}$	$\lVert u\rVert_\infty + [u]_{\alpha,\frac{\alpha}{2};Q_T}$
$\lvert u\rvert_{2+\alpha,1+\frac{\alpha}{2};Q_T}$	$\lVert u\rVert_\infty + [\partial_t u]_{\alpha,\frac{\alpha}{2};Q_T} + [D^2 u]_{\alpha,\frac{\alpha}{2};Q_T}$
$W \hookrightarrow H$	the space W is continuously embedded in H.
$l^1(\mathbb{R})$	space of sequences $(\lambda_n)_{n\in\mathbb{N}}$ such that $\sum_{n\in\mathbb{N}} \lvert\lambda_n\rvert < \infty$.