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ON THE GEOMETRY OF THE FINSLERIAN ALMOST COMPLEX SPACES

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Sommario. *In questa nota, si studiano gli spazi di Finsler quasicomplessi e gli spazi di Finsler kaehleriani. I metodi usati sono di carattere globale ed utilizzano la complessificazione del fibrato indotto naturalmente dal fibrato tangente e dalla proiezione canonica del sottofibrato dei vettori tangenti non nulli, su una varietà differenziabile di classe* C^∞

1. INTRODUCTION. The complex Finsler structures were first studied by G. Rizza, see [21]. H. Rund has explained the condition of complex homogenity of the foundamental metric function and has obtained the connection coefficients and the equation of geodedics, see [20]. M. Dhawan and M. Chawla have introduced the notion of almost complex Finsler space and kaehlerian Finsler metric. See [3], [4]. A different aproach for the notion of a kaehlerian Finsler space was given by N. Prakash, see [18]. I Ghinea and Gh. Atanasiu have find the set of all Finsle connections compatible with almost complex of hermitian Finsler structures, in the more general sense of [17]. See [2], [10]. Complex Finsler geometry was also proved to be a successfull differential geometric method in studying,

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ample and negative vector bundles, in the sense of algebraic geometry (see [12]) as shown by S. Kobayashi, [14]. Following the technique of H. Akbar-Zadeh and B.T.Hassan, see [1], [13], the authors of the present paper have tried to develop a curvature theory for the kaehlerian Finsler spaces, in a global manner. See [6] , [7].

In the present paper we intend to continue the study of almost complex and kaehlerian Finsler spaces using the complexification of the naturally induced GL(2n,R)-vector bundle $\pi^{-1}T\,M$.

2. ALMOST COMPLEX π-STRUCTURES.

Let M be a real C^{∞}-differentiable connected m-dimensional manifold M. We denote by: $T(M) \to M$ it's tangent bundle and by $\pi : V(M) \to M$ the subbundle of all non-nul tangent vectors on M. Let $\pi^{-1}TM \to V(M)$ be the bundle naturally induced by T(M) and π . See [9] ,[13] for details on the constructions summaried here. A fibre ovex $\tilde{x} \in V(M)$ in $\pi^{-1}TM$ is denoted by $\pi_{\tilde{x}}^{-1}$ TM. A bundle morphism: $J : \pi^{-1}\,T\,M \to \pi^{-1}\,T\,M$ which is an anti-involution, that is $J^2 = -I$, is called an almost complex π-structure on M. Here I denotes the identity transformation on π^{-1} TM. Let (U,x^i) be a coordinate system on M. Every tangent vector field X on M has a natural lift $\bar{X} : V(M) \to \pi^{-1}TM$ defined by: $\bar{X}(\tilde{x}) = (\tilde{x},X(\pi\tilde{x}))$, $\tilde{x} \in V(M)$. Let then $\{\bar{\partial}_i\}$ be the natural lifts of the local tangent vector fields $\{\frac{\partial}{\partial x^i}\}$. As the real vector space structure of $\pi_{\tilde{x}}^{-1}TM$ is induced by that of $T_{\pi(\tilde{x})}(M)$ we conclude that $\{\bar{\partial}_i\}_{\tilde{x}}$ is a linear basis of $\pi_{\tilde{x}}^{-1}$ TM. Hence π^{-1} T M is a vector bundle having R^m as standard fibre and GL(m,R) as structure group. Obviously the m-dimensional manifold M carrying an almost complex π-structure is even dimensional, that is m = 2n. We shall have locally: $J\bar{\partial}_i = J_i^j \cdot \bar{\partial}_j$, where $J_i^j = J_i^j(x,u)$, and $J_i^k \, J_k^j = - \delta_i^j$.

Let $(\pi_{\tilde{x}}^{-1}\ TM)^c$ be the complexification of $\pi_{\tilde{x}}^{-1}TM$, that is $(\pi_{\tilde{x}}^{-1}T\ M)^c = (\pi_{\tilde{x}}^{-1}T\ M) \otimes_R \mathbb{C}$. We obtain a $\mathbb{C}$-vector bundle $(\pi^{-1}TM)^c \to$ $\to V(M)$.

Let J be an almost complex π-structure on M; then J can be easily prolongated to $(\pi^{-1}\ TM)^c$. Obviously J has on real eigen-values. Let $(\pi_{\tilde{x}}^{-1}\ TM)^{1,0}$ and $(\pi_{\tilde{x}}^{-1}\ TM)^{0,1}$ be the eigen-values of J corresponding to the eigen-values i and -i, where $i = \sqrt{-1}$. The following result is obvious now:

Proposition 2.1.

1. $(\pi_{\tilde{x}}^{-1}\ T\ M)^c = (\pi_{\tilde{x}}^{-1}\ TM)^{1,0} + (\pi_{\tilde{x}}^{-1}\ TM)^{0,1}$ (direct sum)

2. $(\pi_{\tilde{x}}^{-1}\ TM)^{1,0} = \{\bar{X} - i\ J\ \bar{X}/\bar{X} \in \pi_{\tilde{x}}^{-1}\ T\ M\}$

3. $(\pi_{\tilde{x}}^{-1}\ TM)^{0,1} = \{\bar{X} + i\ J\ \bar{X}/\bar{X} \in \pi_{\tilde{x}}^{-1}\ T\ M\}$

Cross-sections $\bar{X} : V(M) \to \pi^{-1}\ TM$ are refered to as π-vector fields on M. Also $\bar{Z} \in (\pi^{-1}\ TM)^{1,0}$ is said to be a holomorphic π-vector on M and $\bar{Z} \in (\pi^{-1}\ T\ M)^{0,1}$ is said to be an anti-holomorphic π-vector on M.

Let $T^c(M) \to M$ be the complexification of the tangent bundle $T(M)$ over M, that is $T^c(M) = T(M) \otimes_R \mathbb{C}$. If $p : V(M) \times T^c(M) \to V(M)$ and $\hat{\pi}: V(M) \times T^c(M) \to T^c(M)$ are the natural projections of the product manifold $V(M) \times T^c(M)$, let $\pi^{-1}T^cM$ be the bundle naturally induced by $T^c(M)$ and π, that is, the following diagram is commutative:

$$\begin{array}{ccc} \pi^{-1}\,T^cM & \xrightarrow{\quad p \quad} & V(M) \\ \hat{\pi}\downarrow & & \downarrow \pi \\ T^c(M) & \longrightarrow & M \end{array}$$

Let $(\tilde{x},u) \in \pi^{-1}_{\tilde{x}}T^cM$, $\tilde{x} \in V(M)$, be fixed. Then: $w \in T^c_{\pi x}(M)$, that is $w = u+iv$, $u,v \in T_{\pi\tilde{x}}(M)$. We can build the linear map: $(\tilde{x},u) \mapsto \bar{Z}$, where $\bar{Z} = \bar{X} + i\bar{Y}$, $\bar{X} = (\tilde{x},u)$, $\bar{Y} = (x,v)$. We conclude with the following:

Proposition 2.2.

The complexification $(\pi^{-1}T\,M)^c$ of $\pi^{-1}T\,M$ and $\pi^{-1}T^cM$ are isomorphic vector bundles.

Let M be a complex manifold of complex dimension n. We proceed to make the notations clear for all local computations. Let (z^α) be the complex coordinates of M, $\alpha = 1,2,\dots,n$. If we put: $z^\alpha = x^\alpha + iy^\alpha$, $\alpha = 1,2,\dots,n$, then M becomes a real analytic manifold with the coordinate (x^α,y^α). If we put $x^{\bar{\alpha}} = y^\alpha$ then we denote (x^α,y^α) by (x^k), $k = 1,2,\dots n,\bar{1},\bar{2},\dots,\bar{n}$. Then the values of $\{\frac{\partial}{\partial x^k}\} = \{\frac{\partial}{\partial x^\alpha}, \frac{\partial}{\partial y^\alpha}\}$ at $x \in M$ give a linear basis of $T_x(M)$. Let $\{\bar\partial_k\} = \{\bar\partial_\alpha, \bar\partial_{\bar\alpha}\}$ be the natural lifts of $\{\frac{\partial}{\partial x^k}\}$. Then M admits a natural almost complex π-structure: $J:\pi^{-1}TM \to \pi^{-1}TM$, $J\bar\partial_\alpha = \bar\partial_{\bar\alpha}$, $J\bar\partial_{\bar\alpha} = \bar\partial_\alpha$, $\alpha = 1,2,\ldots,n$.

Let us put also:

$$\frac{\partial}{\partial z^\alpha} = \frac{1}{2}\left(\frac{\partial}{\partial x^\alpha} - i\frac{\partial}{\partial y^\alpha}\right)$$

$$\frac{\partial}{\partial z^{*\alpha}} = \frac{1}{2}\left(\frac{\partial}{\partial x^\alpha} + i\frac{\partial}{\partial y^\alpha}\right)$$

If we denote $\frac{\partial}{\partial z^{*\alpha}}$ by $\frac{\partial}{\partial z^{\bar\alpha}}$ we can denote $\{\frac{\partial}{\partial z^\alpha}, \frac{\partial}{\partial z^{\bar\alpha}}\}$ by $\{\frac{\partial}{\partial z^k}\}$, $k = 1,2,\ldots,n,\bar1,\bar2,\ldots,\bar n$. It is known that $\{\frac{\partial}{\partial z^\alpha}\}_x$ is a basis for the space of holomorphic vector $T_x^{1,0}(M)$ and $\{\frac{\partial}{\partial z^\alpha}\}_x$ is a basis for the space of anti-holomorphic vectors $T_x^{0,1}(M)$, where $x \in M$. In an analogous manner, let us put:

$$\frac{\bar\partial}{\partial z^\alpha} = \frac{1}{2}(\bar\partial_\alpha - 1\,\bar\partial_{\bar\alpha})$$

$$\frac{\bar\partial}{\partial z^{\bar\alpha}} = \frac{1}{2}(\bar\partial_\alpha + i\,\bar\partial_{\bar\alpha})$$

We obtain a local basis for the space of holomorphic π-vectors, respectively for the space of anti-holomorphic π-vectors.

Let (x^i,u^i) be the coordinates on T(M) and V(M) naturally induced by (U,x^i). If we put $(u^i) = (u^\alpha,u^{\bar\alpha})$, where: $i=1,2,\dots,n,\bar 1,\bar 2,\dots,\bar n$, $\alpha = 1,2,\dots,n$, and $v^\alpha = u^{\bar\alpha}$, and $\dot z^\alpha = u^\alpha + i\cdot v^\alpha$, then T(M) and V(M) have as well the coordinates $(z,\dot z)$. Then $\{\frac{\partial}{\partial x^k}, \frac{\partial}{\partial u^k}\}_{\tilde x}$ is a basis for $T_{\tilde x}(V(M))$ and $\{\frac{\partial}{\partial z^k}, \frac{\partial}{\partial \dot z^k}\}_{\tilde x}$ is a basis for $T^c_{\tilde x}(V(M))$. Here we have denoted: $\frac{\partial}{\partial z^k} = \{\frac{\partial}{\partial z^\alpha}, \frac{\partial}{\partial z^{\bar\alpha}}\}$,

$$\{\frac{\partial}{\partial \dot z^k}\} = \{\frac{\partial}{\partial \dot z^\alpha}, \frac{\partial}{\partial \dot z^{\bar\alpha}}\}, \text{ where: } \frac{\partial}{\partial \dot z^{\bar\alpha}} = \frac{\partial}{\partial \dot z^{*\alpha}}$$

and:

$$\frac{\partial}{\partial \dot z^\alpha} = \tfrac12 \left(\frac{\partial}{\partial u^\alpha} - i\frac{\partial}{\partial v}\right)$$

$$\frac{\partial}{\partial \dot z^{*\alpha}} = \tfrac12 \left(\frac{\partial}{\partial u^\alpha} + i\frac{\partial}{\partial v^\alpha}\right)$$

Also we denote, for simplicity:

$$\frac{\partial}{\partial x^k} = \partial_k \quad , \quad \frac{\partial}{\partial u^k} = \partial_{,k}$$

$$\frac{\partial}{\partial z^k} = \partial^c_k \quad , \quad \frac{\partial}{\partial \dot z^k} = \partial^c_{,k}$$

3. SELF-ADJOINT TENSOR FIELDS.

Let $\tilde{x} \in V(M)$ be fixed. A multillinear map:

$\bar{K}_{\tilde{x}} : \pi_{\tilde{x}}^{-1} T M \times \ldots \pi_{\tilde{x}}^{-1} T M \to R$, where the direct product has s factors is said to be a π-tensor of type (0,s). Then $\bar{K} : \tilde{x} \mapsto \bar{K}_{\tilde{x}}$ is said to be a π-tensor field of type (0,s), and we can write:

$\bar{K} \in (\pi^{-1} T M)^* \otimes \ldots \otimes (\pi^{-1} T M)^*$, where $(\pi^{-1} T M)^*$ denotes the dual vector bundle of $\pi^{-1} T M$.

We denote by $\Lambda^s(\pi,M)$ the space of all π-tensor fields of type (0,s) which are skew-symmetric. They are refered to as π-forms of de gree s. Let $\Lambda^s(M) \to M$ be the vector bundle of all differentiable s-forms on M. Let then $x^{-1}\ \Lambda^s(M) \to V(M)$ be the bundle naturally induced by $\Lambda^s(M)$ and π. Let $\bar{\omega} \in \Lambda_x^s(\pi,M)$ be fixed. We define an element: $\bar{\omega}' \in \Lambda_x^{-1}\ {}^s(M)$ by $\bar{\omega}'=(x,\pi)$, where $\pi(X_1,X_2,\ldots,X_s) = \bar{\omega}(\bar{X}_1,\bar{X}_2,\ldots,\bar{X}_s)$ where $\bar{X}_i = (x,X_i)$, $i=1,2,\ldots,s$, for any $X_1,X_2,\ldots,X_s \in T_x(M)$. Hence $\bar{\omega} \to \bar{\omega}'$ gives an izomorphism of $\Lambda_x^s(\pi,M)$ on-to $\pi_x^{-1}\ \Lambda^s(M)$. We conclude with the following:

Proposition 3.1.

$\Lambda^s(\pi,M)$ and $\pi^{-1}\ \Lambda^s(M)$ are izomorphic vector bundles.

Hence a cross-section $\bar{\omega} : V(M) \to \pi^{-1}\ \Lambda^s(M)$ is exactly a π-form of decree s. Similar constructions were done in [11] for π-forms of degree 1. Using the izomorphism given by prop. 4.1. it follows that a π-tensor field of type (0,s) is an element $\bar{K} \in \pi^{-1} T^*M \otimes \ldots$ $\ldots \otimes \pi^{-1} T^*M$. We denote by $\mathcal{T}_{r,s}(\pi,M) = \pi^{-1} T^*M \otimes \ldots \otimes \pi^{-1} T^*M \otimes$

$\otimes \pi^{-1} T M \otimes \ldots \otimes \pi^{-1} T M$, the space of all π-tensor fields of type (r,s). Then let $\mathcal{T}^c_{r,s}(\pi,M)$ be the complexification of $\mathcal{T}_{r,s}(\pi,M)$, that is $\mathcal{T}^c_{r,s}(\pi,M) = \mathcal{T}_{r,s}(\pi,M) \otimes \mathbb{C}$. We define the notion of self-adjoint π-tensor field for the case of the π-tensor fields of type $(1,2)$, for the sake of simplicity.

Let $\bar{K} \in \mathcal{T}^c_{r,s}(\pi,M)$ be fixed; then $\bar{K}$ can be regarded as a morphism: $\bar{K} : \pi^{-1}T^cM \times \pi^{-1}T^cM \to \pi^{-1}T^cM$. Hence we can put

$\bar{K}\left(\frac{\bar{\partial}}{\partial z^k}, \frac{\bar{\partial}}{\partial z^j}\right) = K^m_{kj} \cdot \frac{\bar{\partial}}{\partial z^m}$ and $\bar{K}$ has various components

$$K^{\alpha}_{\beta\gamma}\ ,\quad K^{\alpha}_{\bar{\beta}\gamma}\ ,\quad K^{\alpha}_{\bar{\beta}\bar{\gamma}}$$

$$K^{\bar{\alpha}}_{\beta\gamma}\quad K^{\bar{\alpha}}_{\bar{\beta}\gamma}\quad K^{\bar{\alpha}}_{\bar{\beta}\bar{\gamma}}$$

We define the adjoint $A(\bar{K})$ of $\bar{K}$ by: $A(\bar{K})\left(\frac{\bar{\partial}}{\partial z^k}, \frac{\bar{\partial}}{\partial z^j}\right) = K^{\bar{m}}_{\bar{k}\bar{j}} \cdot \frac{\bar{\partial}}{\partial z^m}$, where $\bar{k} = \alpha$, if $k = \bar{\alpha}$, and $\bar{k} = \bar{\alpha}$ if $k = \alpha$.

Then $\bar{K}$ is said to be self-adjoint if $A(\bar{K}) = \bar{K}^*$; here the star denotes the complex conjugation. That is, $\bar{K}$ is said self-adjoint if barring and unbarring all indices simultaneously the values of the component changes into it's complex conjugate.

4. HERMITIAN FINSLER SPACES.

Let M be a 2 n-dimensional differentiable connected manifold and let J be an almost complex π-structure on M.

Let $E : V(M) \to R_+$ be a Finsler energy on M, that is a C^∞-differentiable function on V(M) having the following properties:

1. $E \in C^1(T(M))$, if extended to be zero on the zero-section.

2. E is positively-homogeneous of degree 2, that is: $E(kv) = k^2 E(v)$, for any $k > 0$, $v \in V(M)$.

3. The quadratic form: $g_{ij}(x,u) = \frac{1}{2} \frac{\partial^2 E}{\partial u^i \partial u^j}$ is positive-definite.

The Finsler energy E on M induces naturally a metric π-tensor field $g \in \pi^{-1}T^*M \otimes \pi^{-1}T^*M$. Then (M,E,J) is said to be *an almost complex Finsler space*. The almost complex Finsler space (M,E,J) is said to be *a hermitian Finsler space* if: $g(J\bar{X},J\bar{Y}) = g(\bar{X},\bar{Y})$ for any π-vector fields $\bar{X},\bar{Y}$ on M. Then g can be easily extended to a hermitian inner product on $\pi^{-1}T^cM$, that is:

$$(4.1) \qquad g(\bar{Z}^* , \bar{W}^*) = g(\bar{Z} , \bar{W})^*$$

for any complex π-vector fields $\bar{Z}$, $\bar{W}$ on M.

$$g(\bar{Z} , \bar{Z}^*) > 0$$

for all non-zero complex π-vector field $\bar{Z}$ on M.

$$g(\bar{Z} , \bar{W}^*) = 0$$

for any holomorphic π-vector field $\bar{Z}$ on M and any anti-holomorphic π-vector field $\bar{W}$ on M.

The fundamental π-form is a π-form of degree 2 on M defined by $\bar{\emptyset}(\bar{X} , \bar{Y}) = g(\bar{X},J\bar{Y})$, for any π-vector fields $\bar{X}$, $\bar{Y}$ on M. Then $\bar{\emptyset}$ can be easily extended to $\pi^{-1}T^cM$. Obviously $g(\bar{X},\bar{Y}) = 0$,

$\bar{\phi}(\bar{X}, \bar{Y}) = 0$ if $\bar{X}$, $\bar{Y}$ are both holomorphic or both anti-holomorphic π-vector fields on M. Hence we have:

$$(4.2)\qquad g_{\alpha\beta} = 0 \quad , \quad g_{\bar{\alpha}\bar{\beta}} = 0$$

$$\phi_{\alpha\beta} = 0 \quad , \quad \phi_{\bar{\alpha}\bar{\beta}} = 0$$

Using (4.1) and (4.2) we can establish the following:

Proposition 4.1.

$$g_{\alpha\bar{\beta}} = g^*_{\bar{\alpha}\beta}$$

Proof.

$$g_{\alpha\bar{\beta}} = g\left(\frac{\bar{\partial}}{\partial z^\alpha}, \frac{\partial}{\partial z^\beta}\right) = g\left(\frac{\bar{\partial}}{\partial z^\alpha}, \frac{\partial}{\partial z^{*\beta}}\right) =$$

$$= g\left(\frac{\bar{\partial}}{\partial z^\alpha}, \left(\frac{\bar{\partial}}{\partial z^\beta}\right)^*\right) = g\left(\frac{\bar{\partial}}{\partial z^\alpha}\right)^*, \frac{\bar{\partial}}{\partial z^\beta}\Big)^* = g^*_{\bar{\alpha}\beta} .$$

Let $\pi^{-1}T^{*c}M \to V(M)$ be the complexification of $\pi^{-1}T^*M$, that is $\pi^{-1}T^{*c}M = \pi^{-1}T^*M \otimes \mathbb{C}$. Then let $\{\bar{d}x^k\}$ be the natural lifts of the local 1-forms $\{dx^i\}$ on M. If $\omega : M \to T^*(M)$ is a 1-form on M, then $\bar{\omega} : V(M) \to \pi^{-1}T^*M$ defined by $\bar{\omega}(\tilde{x}) = (\tilde{x}, \omega(\pi\tilde{x}))$, $\tilde{x}\in V(M)$ is said to be the natural lift of ω . Then we can put:

$$\bar{d}\, z^\alpha = \bar{d}x^\alpha + i\,\bar{d}\,y^\alpha$$

$$\bar{d}\, z^{*\alpha} = \bar{d}x^\alpha - i\,\bar{d}\,y^\alpha$$

and denote $\{\bar{d}z^{\alpha}, \bar{d}z^{\bar{\alpha}}\}$, where $\bar{d}z^{\bar{\alpha}} = \bar{d}z^{*\alpha}$, by $\{\bar{d}z^{k}\}$. Then the almost complex π-structure J can be easily extended to $\pi^{-1}T^{*}M$ and hence to $\pi^{-1}T^{*c}M$. If $(\pi^{-1}T M)^{1,0}$, and $(\pi^{-1}T M)^{0,1}$ are the eigen-spaces of J corresponding to the eigen-values i and -i, then we could easily give an analogous of prop. 2.1. Then $\bar{\omega} \in (\pi^{-1}T^{*}M)^{1,0}$ is called a holomorphic π-form of degree 1 and $\bar{\omega} \in (\pi^{-1}T^{*}M)^{0,1}$ an anti-holomorphic π-form of degree 1 on M.

The exterior product of two π-forms of degree 1 could be given as: $(\bar{\omega} \wedge \bar{\theta})(\bar{X}, \bar{Y}) = \frac{1}{2}\{\bar{\omega}(\bar{X})\,\bar{\theta}(\bar{Y}) - \bar{\omega}(\bar{Y})\,\bar{\theta}(\bar{X})\}$, for any π-vector fields $\bar{X}$, $\bar{Y}$ on M.

Proposition 4.2.

$$\emptyset = -2i\, g_{\alpha\bar{\beta}}\, \bar{d}z^{\alpha} \wedge \bar{d}z^{\bar{\beta}}$$

Proof.

The desired result follows by computation using the formulae:

$$\emptyset_{\alpha\bar{\beta}} = -i\, g_{\alpha\bar{\beta}}$$
$$\emptyset_{\bar{\alpha}\beta} = i\, g_{\bar{\alpha}\beta}$$

We have to note that the meaning of prop. 4.1. is that A(g) = g*, that is the Finsler hermitian metric g is self-adjoint. See also [18]. The formalism developed in the present paper will be used in a fortheoming paper in order to make a comparisson between the two existent definitions of a kaehlerian Finsler space, that is the definitions from [3] and from [18]. It is known that they are equivalent in the Riemannian case.

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