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PERIODIC INVENTORY MODEL WITH REDUCED SETUP COST UNDER SERVICE LEVEL CONSTRAINT

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Abstract: *The underlying goal of JIT philosophy is to eliminate waste, which is possible only through certain measures such as reducing the lead time, crashing of the setup cost, improving the service level etc. All these measures are achievable through an extra investment. The present study investigates the effect of crashing of the lead time and setup cost in periodic review inventory model, where the demand during the protection interval follows the normal distribution under the service level constraint. Numerical example is presented to illustrate the results of the proposed model along with the sensitivity analysis.*

Keywords: *Inventory, setup cost, crashing cost, Service level, Lead-time.*

1. Introduction

Just-In-Time (JIT) philosophy includes the successful execution of all manufacturing activities that are required to produce the product and its fast delivery for an end user. On the other front, it helps in continuous improvement of the manufacturing process and in the elimination of waste, which ultimately help the companies to provide better product and services at a lesser cost. The utilization of JIT philosophy emphasizes low stock, high quality, improved services and shortening of lead time / setup cost etc. The main idea is to use the resources in an optimal way to reduce the process time as far as possible. Usually, in any inventory modeling (deterministic and stochastic); lead time and setup cost have their own importance. It has been found that in most of the inventory models, authors have assumed that the lead time is fixed; where as in reality, one can reduce the lead time. Tersine [23] suggested that lead time has different components viz. order preparation, order transit, supplier lead times, delivery lead time and setup time which are reducible up to a certain limit. Owing to this fact, each firm tries different ways

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to reach to their respective customers. One such way could be the reduction of lead time. This not only helps the firm to reach out the customers early, but also they are able to provide better services. Hence, each firm tries to control lead time, of course at an extra cost; by doing so, the firm is able to convert some portion of lost sales into the backlogged one. Researchers have developed several continuous review inventory models to reflect the lead-time as a decision variable (Liao and Shyu [10]; Ben-daya and Raouf [1]; Montgomery [11]; Moon and Choi [12]); Ouyang and Chuang [14]; Wu and Tsai [25]; Pan and Hsiao [18]). Pan and Hsiao [19] investigated the model by considering the case where lead-time crashing cost is given as a function of reduced lead-time and ordered quantities. But these authors have not considered any kind of bound over the expected stock out.

Another important component of using the application of JIT philosophy is the crashing of setup cost. Like lead time, one can very well reduce the setup cost at an extra investment. The most common controllable components of setup cost could be the procedural changes, special equipment acquisition and workers training, use of multipurpose machines etc. In fact, all the extra money spent on controlling these components of the setup cost eventually helps the firm to reduce its total cost in long run. Porteus [20] investigated the impact of capital investment in reducing setup costs in the classical EOQ model. Many researchers (Nori and Sarkar [13]); Kim et al. [7]; Trevino et al [24]; Sarkar and coats [21]; Cheng et al. [2]**;** Chuang et al [4]) have extended the research by explaining the association between the amount of capital investment and setup cost level.

Moreover, in stochastic inventory model, usually one has to maintain the safety stock as a cover against backorders. But excess of it has a direct implication on the running cost of the inventory system. In fact, safety stock is directly proportional to the level of service being offered. Ouyang and Wu [16] considered a continuous inventory model involving variable lead-time with a service level constraint. Ouyang and Chuang [15] provided the periodic review model with variable lead time and service level constraint, where the demand during lead time follows the normal distribution. Ouyang et al. [17] investigated the stochastic inventory model with the reduction of setup cost and lead time for unknown distribution of demand with probabilistic backorder rate. Chu et al [3] considered an improved continuous inventory model with service level and lead time. Lee et al. [8] developed computational algorithm for inventory model with a service level constraint where lead time demand follows the mixture of distributions and backorder rate is considered to be negative exponential. Jha and Shanker [6] investigated two-echelon supply chain inventory model with controllable lead time and service level constraint.

Further, Liang et al. [9] have discussed the results provided by Ouyang and Chuang [15] and provided an improved periodic model by using the alternative approach. In these papers, authors have mainly concentrated on the level of service along with the crashing of lead time.

Now there are certain questions, which may boggle the mind of inventory manager, viz.

(i) What will be the effect of crashing of lead time?
(ii) What are the benefits of crashing of setup cost?
(iii) What would be the effect of crashing on review period?
(iv) How much savings the supplier could fetch using the JIT expertise?

in a periodic review inventory model with a service level constraint.

In order to obtain the answer to these questions, a periodic review inventory model by considering the lead time and setup cost as a function of capital investment under the service level constraint has been formulated. Demand during the protection interval is assumed to be normally distributed. The total cost has been managed by jointly optimizing the review period, the setup cost and the lead time. The results have been found to be very encouraging as the manager is not only able to make significant savings by reduction of lead time and setup cost but it also provides him greater flexibility in arriving at the best solution, for his desired level of service and the available investment.

2. Notation and assumptions

To develop the proposed model, the following notation and assumptions have been used.

2.1 Notations

D : Average demand per year
K : Setup cost per inventory cycle
h : Inventory holding cost per unit per year
R : Target inventory level
β : Fraction of the demand back ordered during stock out period such as $0 \le \beta \le 1$.
L : Length of lead-time
σ : The standard deviation of the protection interval $(T+L)$ demand per unit
T : Length of a review period
X : Protection interval demand which has a $p.d.f.$ f_X with finite mean $D(T+L)$ and standard deviation $\sigma\sqrt{(T+L)}(>0)$
α : Proportion of demands that are not met from stock with $(1-\alpha)$ as the service level where $0<\alpha<1$
A : Safety factor
$E(.)$: Mathematical Expectation
x^+ : Maximum value of x and 0 i.e. $x^+ = Max\{x,0\}$.
EAC : Expected Annual Cost
EAC^w : Least upper bound of expected annual cost

2.2 Assumptions

1. The target level *R*= *Expected demand during the protection interval + safety stock (SS)* where *SS*= *A*(standard deviation of protection interval demand)*. Therefore, $R = D(T+L) + A\sigma\sqrt{T+L}$ where A is the safety factor and depends upon the service level $(1-\alpha)$.
2. The inventory level is reviewed every *T* units of time. A sufficient quantity is ordered up to the target level *R*, and the ordering quantity is arrived after *L* units of time, where *L*<*T*, means there is only one order outstanding in any cycle.
3. The lead-time *L* consists of *n* mutually independent components. The i^{th} component has a minimum duration a_i and normal duration b_i , and a crashing cost per unit time c_i, such

that $c_1 \le c_2 \le c_3 .... \le c_n$. Therefore, one starts crashing the lead time from its first component as it has got the minimum unit crashing cost, and then component 2 and so on.

4. Let $L_0 = \sum_{j=1}^{n} b_j$ and L_i be the length of lead time with components $1,\ 2,...,\ i$ crashed to their minimum duration, then L_i can be expressed as $L_i = L_0 - \sum_{j=1}^{i}\left(b_j - a_j\right),\ i = 1, 2, ...,\ n$ and the lead time crashing cost per cycle $C\left(L_i\right)$ is given as $C\left(L_i\right) = c_i\left(L_{i-1} - L\right) + \sum_{j=1}^{i-1} c_j\left(b_j - a_j\right)$, where $L \in (L_{i,} L_{i-1})$ (Ouyang and Chuang [15]).
5. We assumed a service level constraint instead of a stock-out cost term in the total cost function as it is difficult to compute the stock out cost in the inventory system.

3. Model formulation

In a periodic review inventory model, the time between reviews, T represents the time between the arrivals of two successive orders and at each review time, a sufficient quantity is ordered to bring the inventory position up to a level R. The inventory position of the system is R after reviewing and placing an order and an order will arrive in the system after a time lag L (lead time). The expected net inventory immediately after the arrival of procurement is then ($R - DL$), where DL is the expected lead time demand. If the mean rate of demand is constant, the expected on hand inventory at the beginning of the cycle will be ($R - DL$), which decreases to ($R - DL - DT$) just before the arrival of next order. Thus, the average inventory per year is given as $(R - DL - DT/2)$.

Now, in order to compute the average annual cost of backorders where the procurement lead time is L, consider an order is placed at time t, and then it will arrive in the system at time$(t + L)$. The next procurement will arrive in the system at time $(t + L + T)$. The expected number of backorders occurring between $(t + L)$ and $(t + L + T)$ is given by Hadley and Whitin [5]:

$$E(X - R)^+ = \int_R^\infty (X - R) f_X \, dX$$

when the demand in the period $(T + L)$ exceeds R.

Thus, one has to consider the effect of demand during the lead time plus one period's demand, as once the order is placed at time t, another order cannot be placed until time $(t + T)$, therefore, protection is needed for the period $(T + L)$.

Since, it has been assumed that the shortages are partially backlogged and partially lost. As β represents the fraction of the demand back ordered during stock out period. Thus, $\beta E(X - R)^+$ are the backlogged units and $(1 - \beta)E(X - R)^+$ are lost. The only difference between the lost sales and backorders models is in evaluation of safety stock. In the lost sale case, the safety stock does not remain the same as in that of backlogged case, in this situation, one needs to carry some additional units ($(1-\beta)E(X-R)^{+}$) in the safety stock. Therefore, the expected net inventory at the beginning of the period is $\left(R-DL+(1-\beta)E(X-R)^{+}\right)$ with the expected net inventory at the end of the period$\left(R-DL-DT+(1-\beta)E(X-R)^{+}\right)$. Thus, the expected holding cost per period is $h\left[R-DL-DT/2+(1-\beta)E(X-R)^{+}\right]$.

Moreover, it is observed that manufacturer / supplier usually brings change in the existing procedure not only to accelerate it but also increase its flexibility. To achieve this manufacturer / supplier tries to reduce the setup cost, which requires additional investment. It is assumed that the crashing of the setup cost is independent of the lead time. We have considered the logarithmic investment function $I(K)$ to represent the relationship between the crashing of setup cost and the investment, as in Ouyang *et al.* [17]:

$$I(K)=\frac{\eta}{m}\ln\left(\frac{K_0}{K}\right) \quad \text{over } K\in(0,K_0],$$

where K_0 is original setup cost, m represents the percentage decrease in setup cost K per dollar increase in investment with η as the cost of capital per year.

Thus, the total expected annual cost (EAC) is the sum of the setup cost, holding cost, lead time crashing cost and the setup crashing cost and is given by:

$$EAC(T,K,L)=\frac{K}{T}+h\left[\frac{DT}{2}+\mathrm{A}\,\sigma\sqrt{T+L}+(1-\beta)E(X-R)^{+}\right]+\frac{C(L_i)}{T}+I(K) \tag{1}$$

Now, our objective is to find the optimal value of setup cost, lead time and the review period which minimizes the total expected cost subject to service level constraint. Therefore, the problem reduced to:

$$Min\ \ EAC(T,K,L)=\frac{K}{T}+h\left[\frac{DT}{2}+\mathrm{A}\,\sigma\sqrt{T+L}+(1-\beta)E(X-R)^{+}\right]+\frac{C(L_i)}{T}+I(K)$$

Subject to: $\frac{E(X-R)^{+}}{D(T+L)}\le\alpha$, $K\in(0,K_0]$

Further, demand, X, during the protection interval, is assumed to follow normal distribution. Therefore, the expected shortages occurring at the end of the cycle is given by (Appendix 1):

$$E(X-R)^{+}=\int_R^{\infty}(X-R)f_X\,dX=\phi(A)-A[1-\Phi(A)],$$

where ϕ and Φ are the standard normal $p.d.f.$ and $c.d.f.$, respectively.

Let $\psi(A)=\phi(A)-A[1-\Phi(A)]$.

Therefore, $E(X-R)^{+}=\sigma\sqrt{(T+L)}\,\psi(A)>0$ (2)

Substituting the value of $E(X-R)^{+}$ from equation (2), equation (1) reduced to:

$$Min\ \ EAC(T,K,L)=\frac{K+C(L_i)}{T}+h\left[\frac{DT}{2}+\mathrm{A}\,\sigma\sqrt{T+L}+(1-\beta)\sigma\sqrt{(T+L)}\,\psi(A)\right]+\frac{\eta}{m}\ln\left(\frac{K_0}{K}\right)$$

subject to: $\dfrac{\sigma\,\psi(A)}{D\sqrt{(T+L)}}\le\alpha\,,\quad K\in(0,K_0]$ (3)

For fixed T and K, the necessary and sufficient conditions for finding the value of lead time are:

$$\frac{\partial EAC(T,K,L)}{\partial L}=0 \text{ and } \frac{\partial^2 EAC(T,K,L)}{\partial L^2}<0$$

which implies:

$$\frac{\partial EAC(T,K,L)}{\partial L}=-\frac{c_i}{T}+\frac{hA\sigma}{2}(T+L)^{-1/2}+\frac{(1-\beta)h\sigma\psi\,(A)}{2}(T+L)^{-1/2}$$

and

$$\frac{\partial^2 EAC(T,K,L)}{\partial L^2}=-\frac{hA\sigma}{4}(T+L)^{-3/2}-\frac{(1-\beta)h\sigma\psi\,(A)}{4}(T+L)^{-3/2}<0$$

So, $EAC(T,K,L)$ is a concave function of $L\in(L_{i,}L_{i-1})$. Therefore, for fixed T and K, the minimum total expected annual cost will occur at the end points of the interval $(L_{i,}L_{i-1})$.
Now, for fixed $L\in(L_{i,}L_{i-1})$, the necessary conditions for finding the solution of T and K are:

$$\frac{\partial EAC(T,K,L)}{\partial T}=0\ \text{, and } \frac{\partial EAC(T,K,L)}{\partial K}=0$$

We have, $\dfrac{\partial EAC(T,\pi_x,L)}{\partial T}=0\Rightarrow\ \ h\left[\dfrac{D}{2}+\dfrac{\mathrm{A}\sigma}{2}\dfrac{1}{\sqrt{(T+L)}}+(1-\beta)\dfrac{\sigma}{2}\dfrac{1}{\sqrt{(T+L)}}\ \psi(A)\right]=\dfrac{K+C(L_i)}{T^2}$ (4)

and

$$\frac{\partial EAC(T,K,L)}{\partial K}=0\Rightarrow\ K=\frac{\eta T}{m} \tag{5}$$

For a given value of $L\in(L_{i,}L_{i-1})$, $EAC(T,K,L)$ may not necessarily be a global convex function in T (Liang et al.[9]). Therefore, for normally distributed model, we follow the algebraic approach as *Liang et al.* [9]:

$$\mu_i=K_i+C(L_i),\quad \delta_1=(Dh/2),\quad \delta_2=(h\sigma/2)(A+(1-\beta)\psi(A))$$

and define

$$f_i(T) = \delta_1 T^2 + \delta_2 \frac{T^2}{\sqrt{T+L}} - \mu_i \text{ for } i = 0,1,\ldots,n \tag{6}$$

If $f_i(T) = 0 \Rightarrow (Dh/2)T^2 + (h\sigma/2)(A + (1-\beta)\psi(A))\dfrac{T^2}{\sqrt{T+L}} - [K_i + C(L_i)] = 0$,

$$\Rightarrow (Dh/2) + (h\sigma/2)[A + (1-\beta)\psi(A)]\frac{1}{\sqrt{T+L}} = \frac{K_i + C(L_i)}{T^2}$$

which is same as equation (4) i.e. $\dfrac{\partial EAC}{\partial T} = 0$.

Since $f_i(T)$ is an increasing function of T, which increases from $f_i(0) = -\mu_i < 0$ to $f_i(\infty) = \infty$ as $\dfrac{d\, f_i(T)}{dT} > 0$, therefore, by theory of equations, we know that there exist at least one positive value for review period, T for $f_i(T) = 0$, and represented by $\hat{T}_i$. The value of $\hat{T}_i$, so obtained is independent of constraints i.e. $L \le T$ and $\left(\dfrac{\sigma\psi(A)}{D\sqrt{(T_i + L_i)}}\right) \le \alpha$. Now, taking into account all these constraints, the optimal value of review period can be obtained as:

$$T_i^* = \max\left\{\hat{T}_i,\, L_i,\, \left(\frac{\sigma\,\psi(A)}{D\alpha}\right)^2 - L_i\right\} \tag{7}$$

In order to find the optimal value of T, K and L, the following solution procedure has been adopted.

3.1 Solution Procedure

- <u>*Step 1*</u>: For a given value of α, obtain the relative service level as $(1-\alpha)$ and calculate the respective safety factor, A and obtain the value of $\psi(A)$ from the normal table (Silver and Peterson [22]).
  For known value of A, $\psi(A)$ and L_i, $i = 0,1,2,\ldots,n$, Perform <u>*Step2*</u> – <u>*Step4*</u>

- <u>*Step 2*</u>:
  i. Start with $K_i = K_0$
  ii. Substitute K_i to obtain $\hat{T}_i$ from the equation (6) such that $f_i(T)$=0. Then, substitute the value of $\hat{T}_i$ in equation (7) to obtain T_i^*.
  iii. Use T_i^* to find K_i from the equation (5).

- <u>*Step 3*</u>: Compare K_i and K_0
  If $K_i \le K_0$ then K_i is feasible, go to step 4. Otherwise set $K_i = K_0$ and evaluate corresponding value of $\hat{T}_i$ and T_i^* and then, go to step 4.

- *Step 4*: For each $\left(T_i^*, K_i, L_i\right)$, compute the corresponding expected total annual cost. By this process, a number of gradually improved feasible solutions can be obtained. For the optimal solution, repeat sub steps (ii) and (iii) of (Step 2) until the values of K_i and T_i^* remains the same, then go to step 3.

- *Step 5*: $\underset{i=0,1,2\ldots n}{Min}\ EAC\left(T_i^*, K_i, L_i\right)$ provides the optimal solution.

3.2 Special Case

When there is no crashing of setup cost i.e. $I(K)=0$, then the equation (1) is reduced to Liang [9] model:

$$Min\ \ EAC(T,K,L)=\frac{K}{T}+h\left[\frac{DT}{2}+\mathrm{A}\,\sigma\sqrt{T+L}+(1-\beta)E(X-R)^+\right]+\frac{C(L)}{T},$$

Subject to: $\dfrac{\sigma\,\psi(A)}{D\sqrt{(T+L)}}\le\alpha$, $\quad K\in\left(0,K_0\right]$

4. Numerical Example

In order to illustrate the model, an inventory system with the following data has considered: $D=600$ units per year, $K=\$300$ per order, $\sigma=7$ units per week, $h=\$20$ per unit per year, $\eta=0.1$ per dollar per year, $m=0.01538\%$. The lead-time data contains the normal duration and minimum duration with the respective crashing cost given in table 1.

Table 1. Lead time data.

Lead time component i	Normal duration (days) b_i	Minimum duration (days) a_i	Unit Crashing cost per day, c_i
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Firstly, the model has been solved for $\alpha=0.02$ (service level = 98% and safety factor $A=2.05$) and backorder ratio, $\beta=1$ (i.e. the complete backlogged case).
The optimal solutions without crashing are:

$L^*=8$ weeks, $K^*=\$300$ and $EAC^*=\$4484.79$.

Now, using the proposed solution procedure, the crashing of lead time and setup cost has been performed and the calculation has been presented in table 2.
The optimal solutions with crashing are:

$L^*=6$ weeks, $K^*=56$ and $EAC^*=\$2220.27$.

Table 2. Solutions set with reduced setup cost, K_i.

i	L_i (in weeks)	K_i	$\hat{T}_i$	T_i^* (in weeks)	R_i	$EAC(.)$
0	8	300.00	10.23	10.23	271.65	**4484.79**
		127.94	6.60	8.00	242.02	3188.24
		82.46	5.26	8.00	242.02	2753.71
		65.81	4.69	8.00	242.02	2574.01
		58.62	4.42	8.00	242.02	2490.57
		55.25	4.29	8.00	242.02	2449.87
		53.60	4.22	8.00	242.02	2429.51
		51.93	4.15	8.00	242.02	2408.61
1	6	300.00	10.26	10.26	245.47	4442.35
		128.28	6.69	6.69	197.50	3113.19
		83.61	5.42	6.00	188.17	2596.09
		67.78	4.90	6.00	188.17	2387.98
		61.28	4.67	6.00	188.17	2297.20
		58.42	4.57	6.00	188.17	2255.98
		57.11	4.52	6.00	188.17	2236.81
		55.99	4.48	6.00	188.17	**2220.27***
2	4	300.00	10.46	10.46	221.46	4440.59
		130.83	7.09	7.09	175.76	3150.64
		88.66	5.99	5.99	160.67	2671.72
		74.93	5.59	5.59	155.15	2483.88
		69.95	5.44	5.44	153.05	2410.05
		68.06	5.38	5.38	152.24	2381.10
		66.86	5.35	5.35	151.72	2362.44
3	3	300.00	10.99	10.99	215.15	4539.99
		137.47	7.99	7.99	174.40	3349.01
		99.92	7.14	7.14	162.70	2956.69
		89.28	6.88	6.88	159.13	2828.52
		86.05	6.80	6.80	158.02	2787.62
		85.05	6.78	6.78	157.68	2774.76
		84.59	6.77	6.77	157.52	2768.80

*optimal solution

From table 2, it can observe that the total expected cost decreases with the fall in setup cost. Further, many options for reduced setup cost are available to the supplier with comparatively lower length of the review period, which implies that the reduced setup cost increases the capabilities of the supplier to manage the inventory more efficiently. The optimal value of lead time is 6 weeks, means we are able to reduce it by 2 weeks, which decreases the inventory investment.

Further, table 3 summarizes the optimal results obtained by Liang et al [9] approach which considers only the reduction of lead time. However table 4 presents the comparison of the proposed model with that of Liang et al [9].

Table 3. Optimal Solutions with Liang *et al.* [9] approach.

i	L_i (in weeks)	T_i^* (in weeks)	R_i	$EAC(.)$
0	8	10.23	271.65	3930.70
1	6	10.26	245.47	**3889.99**
2	4	10.46	221.46	3901.03
3	3	10.99	215.16	4032.61

Table 4: Savings by the reduction of setup cost in comparison to Liang *et al.* [9].

i	L_i (in weeks)	$\alpha = 0.02\, and\ A = 2.05$, Service level of 98%		
		$EAC(*)$ - Liang et al [9] Model	$EAC(.)$- Present model	Savings (%)
0	8	$3930.70	$2408.61	38.72
1	6	3889.99	2220.27	42.92
2	4	3901.03	2362.44	39.44
3	3	4032.61	2768.80	31.34

$Savings\ \% = \{(EAC(*) - EAC(.))/\ EAC(*)\} \times 100\%$

It is clearly evident from the table 4 that the significant savings (i.e. 30%-40% approx.) can be achieved by the supplier when he jointly optimizes the lead time and setup cost. Further, sensitivity analysis has also been performed with respect to α i.e. stock out risk. Results clearly indicate (table 5) that as α decreases (i.e. service level increases) the target inventory level as well as the total expected cost increases because higher the level of service, higher is the safety stock.

Table 5: Effects of α on different parameters with reduced setup cost

		α= 0.20				α= 0.05			
i	L_i (in weeks)	K_i	T_i^*	R_i	$EAC(.)$	K_i	T_i^*	R_i	$EAC(.)$
0	8	62	8.00	208	1797.14	55	8.00	231	2198.62
1	6	66	6.00	159	**1725.24**	59	6.00	178	**2048.92**
2	4	78	6.21	137	1936.37	70	5.63	147	2218.54
3	3	97	7.70	144	2317.99	88	7.07	153	2617.07
		α= 0.02				α= 0.01			
i	L_i (in weeks)	K_i	T_i^*	R_i	$EAC(.)$	K_i	T_i^*	R_i	$EAC(.)$
0	8	52	8.00	242	2408.61	50	8.00	250	2553.26
1	6	56	6.00	188	**2220.27**	54	6.00	195	**2338.82**
2	4	67	5.35	152	2362.44	65	5.17	155	2460.22
3	3	85	6.77	158	2768.80	82	6.57	161	2871.68

5. Conclusion

The main objective of this paper is to explore the benefits of employing Just-In-Time philosophy viz. the reduction of the lead-time and setup cost in periodic inventory model with service level constraint, when protection interval demand is normally distributed. This study not only provides the greater flexibility to the inventory manager to coordinate his inventory in more efficient manner but also makes larger savings in the total expected cost. Infact, the initial investment for reducing the setup cost is higher; but eventually it reduces the total expected cost of the running inventory system. This suggests that the supplier is ultimately benefited largely by workers training, use of latest technology/machinery, improvements in the old procedure etc.

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Appendix 1

If demand, X , during the protection interval, is assumed to follow normal distribution. Then, the expected shortages occurring at the end of the cycle is given by:

$$E(X-R)^{+}=\int_{R}^{\infty}(X-R)f_X\,dX=\int_{DL+A\sigma\sqrt{(T+L)}}^{\infty}\left(X-D(T+L)-A\sigma\sqrt{(T+L)}\right)\frac{1}{\sqrt{2\pi}\left(\sigma\sqrt{(T+L)}\right)}e^{-\left(\frac{X-D(T+L)}{2\sigma\sqrt{(T+L)}}\right)^2}dX$$

by assumption 1.

Substituting $u=\left(\frac{X-D(T+L)}{\sigma\sqrt{(T+L)}}\right)$ then the expected shortages will be:

$$E(X-R)^{+}=\sigma\sqrt{(T+L)}\left(\int_{A}^{\infty}(u-A)\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{u}{2}\right)^2}du\right)$$

By using the special properties of standard normal distribution which are given below:

$$\int_{A}^{\infty}u\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{u}{2}\right)^2}du=\phi(A)\ ,\quad \int_{A}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{u}{2}\right)^2}du=P(u\geq A)$$

and $P(u\geq A)=[1-\Phi(A)]$

Then, $\left(\int_{A}^{\infty}(u-A)\frac{1}{\sqrt{2\pi}}e^{-\left(\frac{u}{2}\right)^2}du\right)=\phi(A)-A[1-\Phi(A)]$ [Silver and Peterson [22]]

where ϕ and Φ are the standard normal $p.d.f.$ and $c.d.f.$, respectively.