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Longevity risk: a methodology for assessing in a Solvency II perspective

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This paper considers the assessment of longevity risk in the context of a longevity indexed life annuities. The framework is set up in a way that accommodates a variety of regulatory regimes such as Solvency II as well as local actuarial practice, attempting to bridge the gap between academia and practice. In the following the authors compare the results obtained in a Solvency II perspective with those obtained with a partial internal model. The predictions contained in both models are compared with the real probabilities in order to evaluate the deviations due to life expectancy improvements.

keywords: Longevity indexed life annuities, Solvency II, Technical provision, CIR model.

1 Introduction

Longevity risk has become an increasingly important risk facing an increasing proportion of the worlds population. Individuals would like to insure against this risk by purchasing life annuity products or other products with lifetime income guarantees. Annuity providers such as life insurance companies are unable to effectively manage aggregate longevity risk and are limited in capacity. Pension plans have increasingly offered defined contribution benefits with the risk of longevity remaining with individuals. In the Solvency II directive longevity risk is described as *the risk of loss, or of adverse change in the value of insurance liabilities, resulting from changes in the level, trend, or volatility of mortality rates, where a decrease in the mortality rate leads to an increase in the value of insurance liabilities.* Hence, the longevity risk is defined as the economic loss stemming

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from an instantaneous, but permanent, decrease in the mortality intensity used for the calculation of technical provisions. For this reason, actuaries have to employ projected life tables incorporating a forecast of future trends of mortality. Clearly, the risk is that the projections of mortality turn out to be incorrect and the annuitants live longer than expected. Different approaches for the construction of the projected tables have been developed until now, for a full report on this subject see Pitacco (2004), but no one turned out to be suitable for the problem solution. The problem is twofold. On the one hand, insurers have to make the annuities market attractive to the insured. At present, the risk borne out by insurers for insurance annuities, which is undoubtedly too high, is reflected in high premiums charged for these products that discourage individuals who are intending to purchase annuities. On the other hand, Solvency II regulation requires the constitution of appropriate margins that are difficult to bear for an insurance company. The Solvency II directive prescribes a standard formula that an insurance business could use to quantify technical provisions. Solvency II directive allows so called internal models as an alternative to the standard formula, as long as the internal model follows the Solvency II principles and is approved by regulator.

To solve the problem of attracting the annuity market and respecting the Solvency II requirements, many insurance companies and pension funds providers focus in the issue of sharing the longevity risk. An ordinary way to solve this problem is through reinsurance, but this method often involves high costs. The securitization provides a viable alternative (Denuit et al., 2007), but unfortunately the longevity bonds are not a very attractive business for investors. Denuit et al. in (2011) use the reduction of annuity periodic payments in a similar way to what happens in the context of securitization. In this way the risk is shared between insurer and insured, but nevertheless we obtain a significant reduction of benefits for the insured. Richards et al. 2014 proposed a very interesting idea based on the quantification of expectation of change in mortality over a one year horizon. Such an approach lies at the heart of the one year value at risk view of the actuarial liability and allows for Sovency II regime for insurers in the European Union. In practice, in a Solvency II perspective, the risk is calibrated using a 99.5% confidence level (Value at Risk measure) on a one-year time horizon.

We try to develop this concept combining a stochastic model for mortality rates approach with a quantile simulation procedure for the short period survival probabilities in order to quantify the risk of the insurance position (Di Lorenzo et al., 2017). Also we compare the results obtained with those of the standard formula with specific attention to the risk of under reserving.

The paper is organized as follows. In Section 1 the longevity index and the longevity indexed life annuity are treated. Section 2 describes the specific features of the Solvency II directive and introduces a coherent internal model for calculating the insurer risk exposure. In Section 3 the issue of modeling the uncertainty in future mortality is fronted and a CIR type model for describing the future evolution of hazard rates is described. Section 4 looks for the conditions that allow to quantify the longevity risk via quantile analysis. Section 5 concludes.

2 Longevity index

Let us consider an individual aged x in the calendar year t. His remaining lifetime is indicated by the notation $T_x(t)$. Therefore, the individual will die at age $x+ T_x(t)$ in the calendar year $t+ T_x(t)$. Then $q_x(t) = P(T_x(t) \leq 1)$ is the probability that an individual aged x in the calendar year t dies before reaching the age x+1 and $p_x(t) = 1 - q_x(t) = P(T_x(t) > 1)$ is the probability that the same individual reaches the age x+1.

Let $p^{mod}_{x+k}(t + k)$ $(k = 0, 1, 2, ..., \omega)$ be the predicted one year survival probability referred to an individual aged x in the calendar year t deducted by some survival model, where ω denotes the ultimate age. Therefore $p^{mod}_{x+k}(t+k)$ is the assumption that is made on the future mortality.

As time passes, the observed values of the one year survival probabilities $p^{obs}_{x+k}(t + k)$ $(k = 0, 1, 2, ..., \omega)$ become available, so that it is possible to compare the values predicted on the basis of a given model with the actual ones, by means of the following ratio:

$$i_{t+k} = \prod_{j=0}^{k-1} \frac{p^{mod}_{x+j}(t+j)}{p^{obs}_{x+j}(t+j)} \tag{1}$$

which can be assessed each future calendar year k. The basic idea is that annual payment due at time k to an individual buying a longevity indexed annuity at age x in calendar year t, is adjusted by the factor (1). Hence, if the contract specifies an annual payment of 1, the annuitant receives a stream of payments $i_{t+1}, i_{t+2}, ...$ as long as he or she survives. In practice, we consider a basic life annuity contract paying one monetary unit of currency at the end of each year as long as the annuitant survives. The single premium is given by:

$$a_x(t) = E\left[\sum_{k=1}^{T_x(t)} 1^{x,k} v(t,k)\right] = \sum_{k=1}^{\omega-x} v(t,k)_k p_x(t) \tag{2}$$

where $1^{x,k}$ is an indicator which equals one if the individual aging x at time t is alive in the future year k $(k = 1, 2, 3...., \omega - x)$, $v(t, k)$ is the deterministic discounting factor.

At this point, if the predictions contained in the model are chosen such that the increasein longevity is greater than predicted, then the payments due to the insured are reduced accordingly. Substantially, the random longevity indexed life annuity, $a^{L.I.}_x$, is given by:

$$a^{L.I.}_x(t) = E\left[\sum_{k=1}^{T_x(t)} 1^{x,k} i_{t+k} v(t,k)\right] = \sum_{k=1}^{\omega-x} v(t,k)_k p_x(t) i_{t+k} \tag{3}$$

3 The Solvency II Directives

Solvency II, by relying on the concept of value at risk, implicitly requires a future forecast in the form of a loss distribution. Further associated concepts such as Market Value Margin require a minimum level of granularity in this distribution, namely, subdivision by calendar period. This directive describes the regulatory requirements to offset the insurance risk for one year risk horizon. In practice, liabilities shall be valued at the amount for which they could be transferred (fair value). The risk of under reserving with respect to longevity is generally captured trough an economic capital model. This model will rely on the distribution of L1 that is the loss random variable for the first calendar year. Generally speaking the value of the technical provision can be described as

$$C = Var_{99.5\%}(L_1) - E(L_1) \tag{4}$$

The value of technical provisions shall be equal to the sum of a best estimate and the capital charge. In Solvency II regulated jurisdictions, there is a non negligible number of companies who are using the standard formula for the determination of the capital charge. The standard formula uses a deterministic shock to the central path of mortality or a Solvency capital requirement based on the Var. In a Solvency II, it is possible to use an internal model as long as this model is follows the Solvency II principles. At this proposal, a coherent structure (Munroe et al., 2015) is given by the following model. The technical provision is the sum of mean loss and the margin discounted to the present value:

$$MVM = (1+i)^{-1} \cdot s \cdot Var_{99.5\%}(L_1) BEL = (1+i)^{-0.5} \cdot s \cdot E(L_1) TP = MVM + BEL \tag{5}$$

where:
MVM=Market Value Margin
BEL=Best Estimate of liabilities
TP= Technical provision
s= spread prescribed by the Solvency II Directive

This model assumes a mortality model plus a forecast. In practice, in its simplest form, a risk model must embody a process that takes as inputs historical data and an instrument that produces loss distributions as output (Munroe et al., 2015). In this way we are able to calculate the loss due to longevity that will arise with a probability of 0.5% on a one-year horizon. This can be calculated as the 99.5% percentile in the loss distribution (due to longevity) obtained from a large number of simulated scenarios for a stochastic model describing the dynamics of the company.

4 The mortality model

Let us consider an individual aged x in the calendar year t. If we consider the hazard rate for an individual aged $x+t$ in the year t μ_{x+t} we have (Bhattacharjee, Misrea):

$${}_kp_x(t) = E[e^{-\int_0^k \mu_{x+s} ds}] \tag{6}$$

We describe the evolution in time of mortality by a widely used stochastic mortality model (Biffis, 2005; Dahl and Moller, 2004), supposing that the force of mortality at time$t$ for an individual aged $x+t$ is given by

$$d\mu_{x+t} = \kappa(\gamma - \mu_{x+t})dt + \sigma\sqrt{\mu_{x+t}}dB_t \tag{7}$$

κ and *sigma* are positive constants, γ is the long term mean and B_t is a Standard Brownian Motion. This model, referred to as the CIR mortality model has the property that the mortality rates are continuous and remain positive.

For convenience, we now introduce the centered version of the model. Let us consider the shifted $\mu^*_{x+t} = \mu_{x+t} - \gamma$. The process is then centred around γ and the long term mean converges almost everywhere to zero:

$$d\mu^*_{x+t} = \kappa\mu^*_{x+t}dt + \sigma\sqrt{\mu^*_{x+t} + \gamma}dB_t \tag{8}$$

with initial condition given by the known value of μ_{x+t}. Its solution is given by

$$E[\mu^*_{x+t}] = e^{-\kappa t}\mu^*_{x+0} \tag{9}$$

$$cov(\mu^*_{x+t}, \mu^*_{x+s}) = \sigma^2\frac{e^{-\kappa t} - e^{-\kappa(s+t)}}{\kappa}\mu^*_{x+0} + \sigma^2\frac{e^{-\kappa(t-s)} - e^{-\kappa(s+t)}}{2\kappa}\gamma, s \leq t \tag{10}$$

$$\lim_{t\to\infty} Var[\mu^*_{x+t}] = \frac{\gamma\sigma^2}{2\kappa} \tag{11}$$

4.1 Parameter estimation procedure

Estimating the parameters of the stochastic mortality model requires the discrete representation of the model. To this aim, we refer to the covariance equivalence principle which requires that the expected values and the stationary variances of the continuous and discrete processes to be equal. The discrete model representation is given by the following equation:

$$\mu^*_{x+t} = \phi\mu_{x+t-1} + \sigma_a\sqrt{\frac{2\phi}{1+\phi}\mu_{x+t-1} + \gamma a_t} \tag{12}$$

The expected value, the covariance and stationary variance functions of the previous equation are:

$$E[\mu^*_{x+t}] = \phi^t\mu^*_{x+0} \tag{13}$$

$$cov(\mu^*_{x+t}, \mu^*_{x+s}) = 2\phi^t\sigma_a^2\mu^*_{x+0}\frac{1-\phi^s}{1-\phi^2} + \phi^{t-s}\sigma_a^2\gamma\frac{1-\phi^{2s}}{1-\phi^2} \tag{14}$$

$$\lim_{t\to\infty} Var[\mu^*_{x+t}] = \frac{\sigma_a^2\gamma}{1-\phi^2} \tag{15}$$

The estimation procedure starts by finding the value of ϕ that minimizes the residual sum of squares function:

$$RSS = \sum_{t=1}^{N}\frac{(\mu^*_{x+t} - \phi\mu^*_{x+t-1})^2}{\frac{2\phi}{1+\phi}\mu^*_{x+t} + \gamma} \tag{16}$$

The least squares estimate of σ_a^2 is given by RSS/N-1. Finally the continuous model parameters are obtained by means of the parametric relationships between continuous and discrete models, derived by applying the covariance equivalence principle:

$$\phi = e^{-\kappa} \tag{17}$$

$$\sigma_a^2 = \sigma^2 \frac{1 - e^{-2\kappa}}{2\kappa} \tag{18}$$

At this point, using the formula proposed by Pitman et al. (1982), we can compute the survival probability

$$_k p_x(t) = E[e^{-\int_0^k \mu_{x+s} ds}] = \frac{exp(-\frac{x}{\sigma^2} w \frac{1+\frac{k}{w} coth(wk/2)}{coth(wk/2)} + (k/w)}{(coth(wt/2) + (k/w) sinhcoth(wk/2))^{\frac{2k\gamma}{\sigma^2}}} \tag{19}$$

where $x = \mu_0$ e $w = \sqrt{k^2 + s\sigma^2}$.
Applying the described estimation procedure, the significant parameters of the mortality-CIR model are obtained and therefore the survival probabilities for each specific calendar year. Our set of data relates to the Italian male population with annual age-specific death counts ranging from ages 64 to 89 over the period 1954 to 2008 (data source: Human mortality database www.mortality.org). We refer to the class of the forward mortality models. These models study changes in the mortality rate curve for a specific age cohorts and capture dynamics of each age cohort over time for all ages greater than x in a specific year t (for example age x in the year *t*, *x+1* in the year *t+1* and so on). In this case, the mortality curves are modeled diagonally (Dahl and Moller, 2004; Cairns et al., 2006; Bauer et al., 2009). In practice, on the basis of data available for the previous 25 years, we can estimate the model parameters for the year t and, as a result, it is possible to get the forecasted survival probabilities. For example, with the data of the period 1954-1978 it is possible to obtain the column of the survival probabilities for the year 1979. This procedure is repeated thirty times in order to obtain the annual survival probabilities over the period 1979 to 2008 and ranging from ages 64 to 89. These probabilities can be compared with the corresponding survival rates obtained from the tables of the Human Mortality Database.
Regarding the choice of fixing the extreme age at 89, recent studies have shown that the most damaging effects in terms of annuities present values for the provider are in the age range 73 to 80.
Clearly this happens because the number of survival is still large at these ages. As a consequence, even modest improvements in the level of survival probabilities with respect to those used for pricing and reserving, result in large additional costs for the annuity provider.
The results of the estimation procedure are summarized in the following table (Tab.1). The parameters κ and σ^2 are obtained, for each year, by means of the relations (7), after the estimation of the discrete parameters in (17)and (18).
We choose to calculate the long term mean γ as the simple mean of each historical series used to estimate the parameters. κ takes the same value for each calendar year. The

reason can be found in the high autoregressive parameter of the discrete model $\phi = 0.999$, which is the same each year explaining the high correlation of each data of each series with the preceding one.

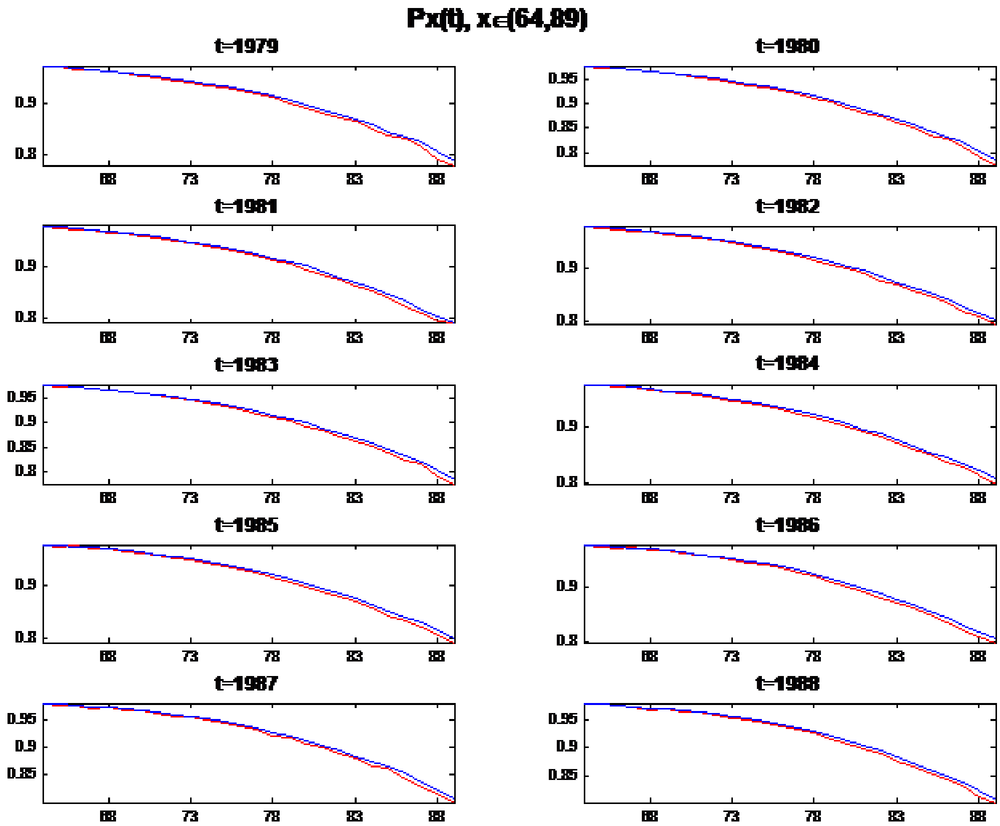

Figure 1: Annual survival probabilities $p_x(t)$ with $x \in (64, 89)$ for each calendar year t ranging from 1979 to 1988. Comparison between CIR model (blue line) and real data (red line).

Figures 1, 2 and 3 show the comparison between the estimated annual survival probabilities obtained by means of the CIR model and the corresponding probabilities of the Italian male population. The results are shown year by year over the period 1979-2008. At this point we model the future uncertainty about mortality by means of the CIR type stochastic process. In practice, the longevity index (1) is computed as:

$$i_{t+k}^{CIR} = \prod_{j=0}^{k-1} \frac{p_{x+j}^{CIR}(t+j)}{p_{x+j}^{obs}(t+j)} \tag{20}$$

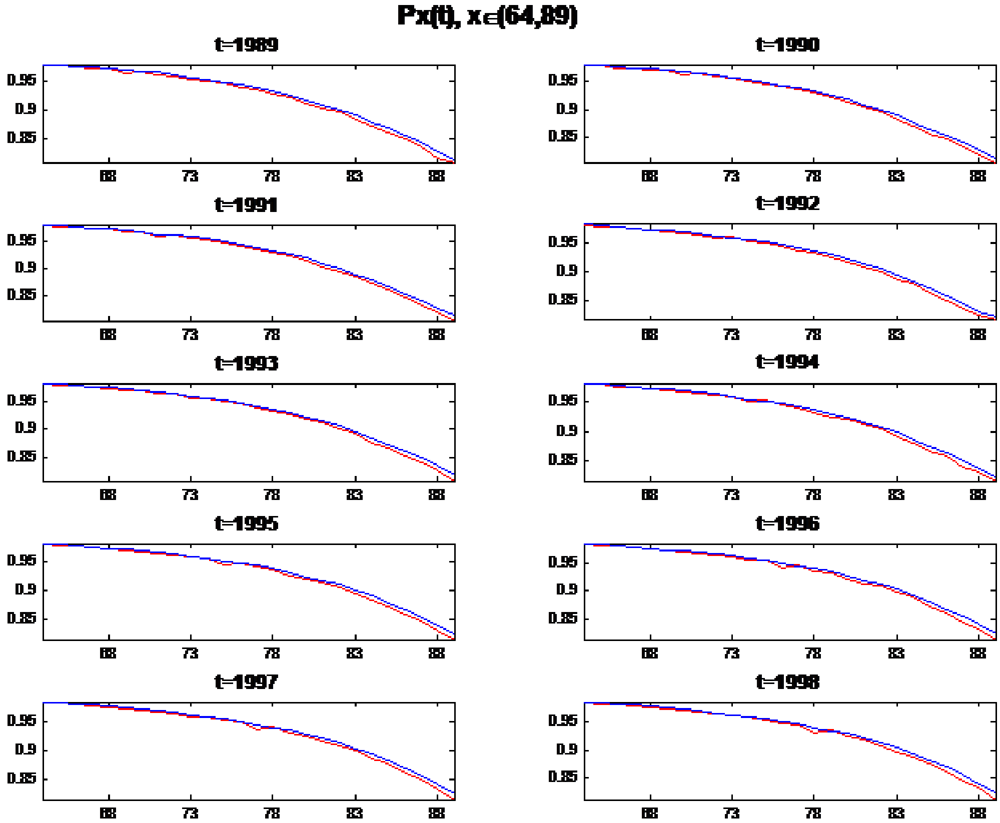

Figure 2: Annual survival probabilities $p_x(t)$ with $x \in (64, 89)$ for each calendar year t ranging from 1989 to 1998. Comparison between CIR model (blue line) and real data (red line).

where:

$p_{x+j}^{CIR}(t+j)$ is a forecast of the annual survival probability of a male aged 64 in 1983. The forecasted probabilities are obtained by means of the CIR type stochastic process on the basis of the parameters estimated; $p_{x+j}^{obs}(t+j)$ are the actual values of the annual survival probabilities deducted from the Italian male mortality tables for the period 1983-2008. In formula (13), $p_{x+j}^{CIR}(t+j)$ are calculated by means of (12), using the estimated parameters for the year 1983, based on the mortality experience of the years 1958-1982.

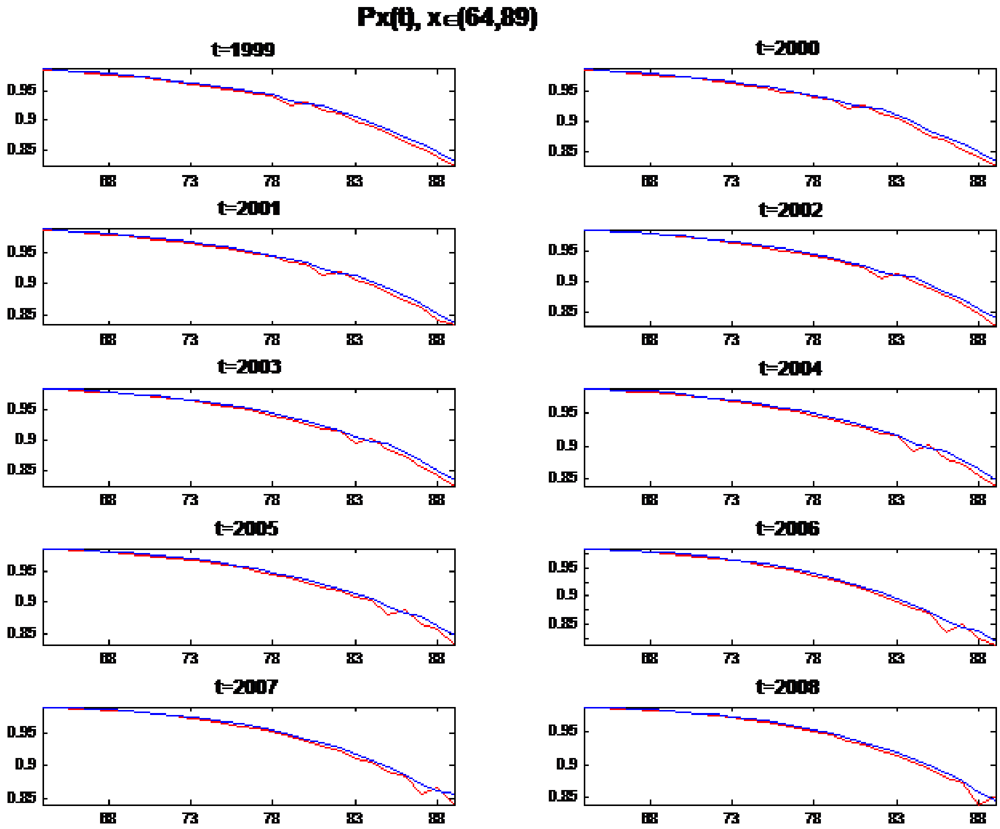

Figure 3: Annual survival probabilities $p_x(t)$ with $x \in (64, 89)$ for each calendar year t ranging from 1999 to 2008. Comparison between CIR model (blue line) and real data (red line).

Fig. 4 shows the comparison between the survival curve estimated by the model and the table available for the year 1983. The choice of the year 1983 can be explained as follows: an indivudual aged 64 in 1983 gets 89 in 2008. Knowing the real data until 2008, the estimated CIR probabilities can be compared with the real data. Now, on the basis of mortality data for the last 25 years, the model is able to provide a good fit to the real survival probabilites of the next year but, unfortunately, fails in projection. In other word, it is not able to capture the decrease in time of the parameters κ and γ because of the well known phenomena of rectangularization and expansion of the Lexis point. For this reason, we try to combine a stochastic model for the evolution of mortality rates with quantile analysis for the mortality distribution in order to capture the trend component of longevity.

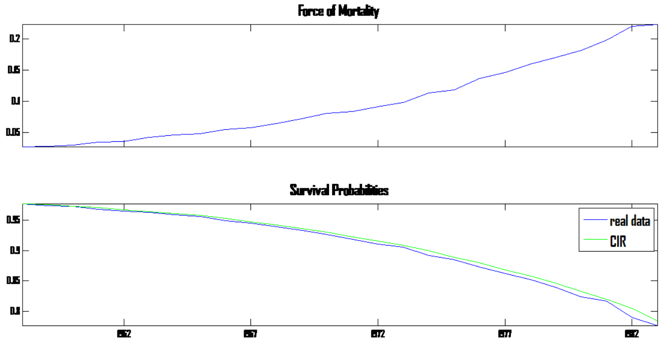

Figure 4: Force of mortality and survival probabilities for an individual aged 64 in 1983 computed by means of the CIR type stochastic mortality model and Italian male mortality table referred to the period 1983 – 2008.

5 The quantile analysis

The quantile estimation gives an important information to the insurer by quantifying the tail events. In our analysis we refer to a tail event as the event of a survival probability higher than the expected one. This is crucial for the insurer. As well as the uncertain phenomena on the life expectancy, it is necessary to quantify the effects due to possible unexpected tail events. Only the awareness of the additional element can help to fully address the longevity risk. We consider the survival probabilities derived by the stochastic model described in the previous section. Fixing the age x we resort to a stochastic simulation procedure and derive, in a one year horizon, a set of cumulative probabilities. Then we estimate the related quantiles. We simulate a large number N of sample paths, each of one producing a simulated set of ${}_tp_x^s(S = 1, , N)$. We set N=10000. The mortality risk measure we refer to is the quantile $MRM = q_\alpha$ where α is the confidence level chosen, in our case the 99.5 per cent. In order to perform the simulation procedure it is necessary to consider the discrete time equation for the chosen Stochastic Differential Equation describing the evolution in time of the mortality rates. On the basis of the first order Euler discretization of equation (8), with a time interval $[x + t, x + t + 1]$ we have:

$$\mu_{x+t_k+1} = \mu_{x+t_k} + \kappa(\lambda - \mu_{x+t_k})\Delta + \sigma\sqrt{\mu_{x+t_k}\Delta}\epsilon_k, k = 1, 2, 3, ..., n - 1 \tag{21}$$

where$\Delta = 1/n$ is the sampling interval , with ϵ_k being the increment ΔB_k of the Wiener process between $t_k = k\Delta$ and $t_{k+1} = (k + 1)\Delta$. The increments ΔB_k are $N(0, \Delta)$ distributed random variables. The discretized process is then represented by the sequence $[\mu_{t_1}, \mu_{t_2}, ..., \mu_{t_n}]$. By means of (6), we obtain the corresponding survival probabilities.

5.1 Longevity risk management via quantile analysis.

The above analysis shows that the model fails to capture the trend of longevity. Although there is a good fit to the data in a year to year perspective, using the probability of the model and comparing them with those available on the tables ex post, you get ratios that are significantly less than one, in the sense that the probability provided by the model are lower than those observed in reality. At this point, we combine the model chosen for the evolution of mortality with a quantile analysis for the mortality distribution. Referring to equation (7), a large number of paths for the force of mortality are simulated. Each path allows to compute a simulated set of probabilities, that is with *x=64*, *t=1979, 1980, 1981, 1982*, and *j=0,1,2,,24*. For each simulated set, we study the longevity index between the simulated survival probability and the *true* probability, that is the probability detected ex post by the life table.
As one can see, the response of the model is good in the medium term. For all the years considered the results of the reports are quite close to 1 up to age 78. The probability of underestimating the survival probabilities is negligible. The situation is different for age greater than 78. In this case the probability of underestimating grows up to 24%. Around this level the probability of underestimation stabilizes even for higher age.

In terms of technical provision for our longevity indexed life annuity, the increase achieved by effect of longevity is at about 23 per cent with respect to the total actuarial liability for the internal model and is at about 21 per cent for the capital charge model. As we can see from the figure the two trends are quite similar. There is also a considerable reduction of 13% in the technical provision with respect to to the case of the 20% deterministic deterioration. These results remains almost constant throughout the period considered are obviously dependent on the insured individuals over the age of 80. At this point it is possible to draw some conclusions. The first is that the combination between a stochastic model for the evolution of the force of mortality and a quantile approach allows to control the deviations of mortality from its expected trend due to the longevity and to limit the probability of underestimation within precise limits. So in our case an increase of the 23 percent reserve limits to 0.5 percent the probability of incurring loss. A reduction can also be noted of the technical provision with respect to the deterministic case. Clearly, if the reserves are used for prudential purposes, the benefits for the insured and the remuneration for the shareholders are limited. Furthermore, in our opinion, the introduction of a single threshold to describe uncontrolled deviations of mortality from its trend looks wrong. The constitutions of the funds by the insurance company should be more responsive to its risk profile. In fact, we see that the effect of longevity is evident in the advanced ages and then it depends effectively on the age classes of the insured.

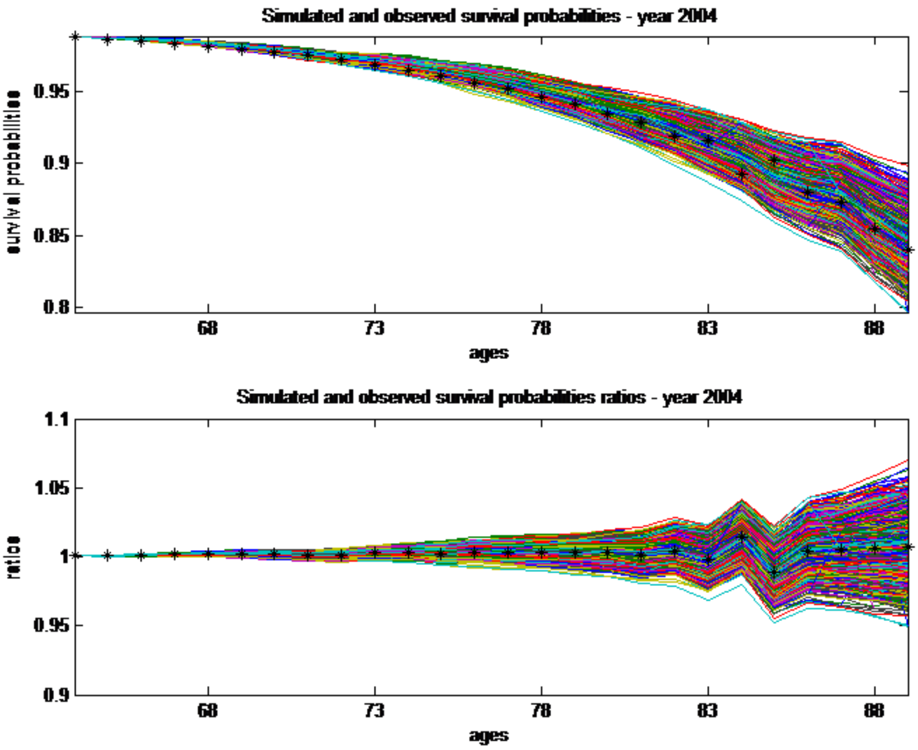

Figure 5: Year 2004. Simulated annual survival probabilities and longevity index. Age ranging from 64 to 89

Table 1: CIR estimated mortality parameters. Data source: Human Mortality Database: Italian male population

year	κ	σ^2	λ
1979	0.0010005	0.02154137	0.09879589
1980	0.0010005	0.02218555	0.09870146
1981	0.0010005	0.02196340	0.09855553
1982	0.0010005	0.02125207	0.09849015
1983	0.0010005	0.02006831	0.09848095
1984	0.0010005	0.02260864	0.09951799
1985	0.0010005	0.02051935	0.09773577
1986	0.0010005	0.02120267	0.09732236
1987	0.0010005	0.01981772	0.09758413
1988	0.0010005	0.01874663	0.09654774
1989	0.0010005	0.01883434	0.09567750
1990	0.0010005	0.01846146	0.09354197
1991	0.0010005	0.01880755	0.09122329
1992	0.0010005	0.01876966	0.09087511
1993	0.0010005	0.01765508	0.09013384
1994	0.0010005	0.01824302	0.08918243
1995	0.0010005	0.01765857	0.08768303
1996	0.0010005	0.01786665	0.08616861
1997	0.0010005	0.01748889	0.08443631
1998	0.0010005	0.01756343	0.08338616
1999	0.0010005	0.0182456	0.08166418
2000	0.0010005	0.01765565	0.07982632
2001	0.0010005	0.0170472	0.07782871
2002	0.0010005	0.01693314	0.07627881
2003	0.0010005	0.01732366	0.07500763
2004	0.0010005	0.01858599	0.07417957
2005	0.0010005	0.01731115	0.07317319
2006	0.0010005	0.01812203	0.07046889
2007	0.0010005	0.01662066	0.06851038
2008	0.0010005	0.01709801	0.06566561

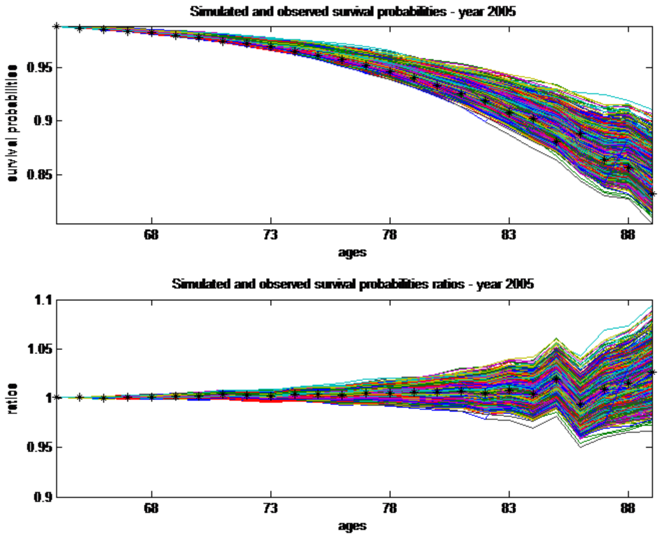

Figure 6: Year 2005. Simulated annual survival probabilities and longevity index. Age ranging from 64 to 89

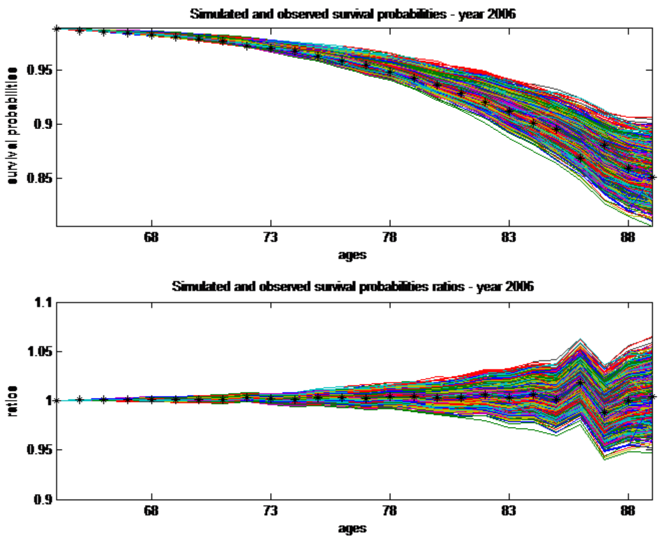

Figure 7: Year 2006. Simulated annual survival probabilities and longevity index. Age ranging from 64 to 89

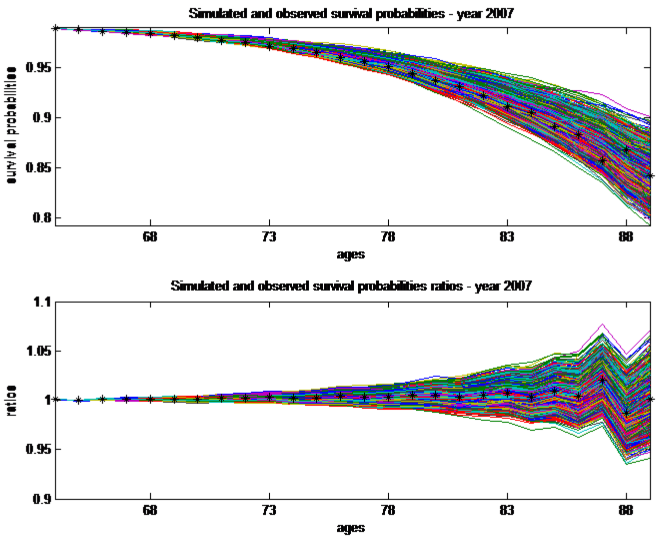

Figure 8: Year 2007.Simulated annual survival probabilities and longevity index. Age ranging from 64 to 89

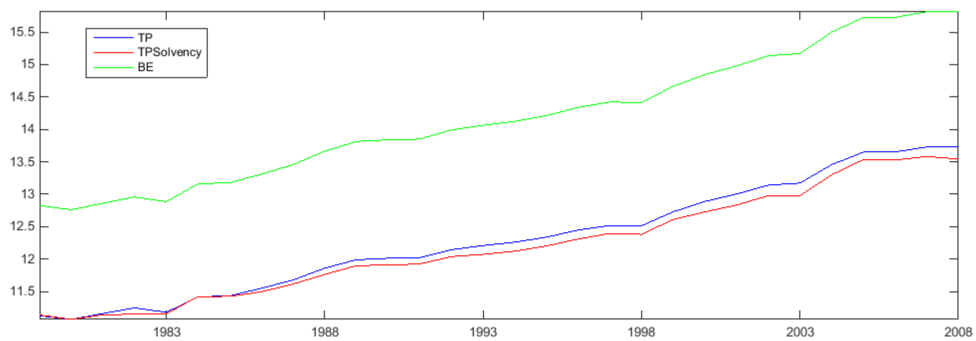

Figure 9: Comparison of technical provisions.

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