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A new count data model with application in genetics and ecology

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The present paper introduces a new count data model which is obtained by compounding negative binomial distribution with Kumaraswamy distribution. The proposed model has several properties such as it can be nested to different compound distributions on specific parameter setting. Similarity of the proposed model with existing compound distribution has been shown by means of reparameterization. Factorial moments and parameter estimation through maximum likelihood estimation and method of moment have been disused. The potentiality of the proposed model has been tested by chi-square goodness of fit test by modeling the real world count data sets from genetics and ecology.

keywords: Negative binomial distribution, Kumaraswamy distribution, compound distribution, factorial moment, count data.

1 Introduction

From the last few decades researchers are busy to obtain new probability distributions by using different techniques such as compounding, T-X family, transmutation etc. but compounding of probability distribution has received maximum attention which is an innovative and sound technique to obtain new probability distributions. The compounding of probability distributions enables us to obtain both discrete as well as continuous distribution. Compound distribution arises when all or some parameters of a distribution known as parent distribution vary according to some probability distribution called the compounding distribution for instance negative binomial distribution can be obtained

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from Poisson distribution when its parameter follows gamma distribution. If the parent distribution is discrete then resultant compound distribution will also be discrete and if the parent distribution is continuous then resultant compound distribution will also be continuous i,e. the support of the original (parent) distribution determines the support of compound distributions.

In several research papers it has been found that compound distributions are very flexible and can be used efficiently to model different types of data sets. With this in mind many compound probability distributions have been constructed. Sankaran (1970) obtained a compound of Poisson distribution with that of Lindley distribution, Zamani and Ismail (2010) constructed a new compound distribution by compounding negative binomial with one parameter Lindley distribution that provides good fit for count data where the probability at zero has a large value. The researchers like Adil Rashid and Jan obtained several compound distributions for instance, Rashid and Jan (2013, 2014b) obtained a compound of zero truncated generalized negative binomial distribution with generalized beta distribution, they obtained compound of a Geeta distribution with generalized beta distribution. Rashid and Jan (2014a) explored a compound of Consul distribution with generalized beta distribution that embodies several compound distributions as sub cases. Based on the same compounding mechanism Rashid et al. (2014) constructed a mixture of generalized negative binomial distribution with that of generalized exponential distribution which contains several compound distributions as its sub cases and proved that this particular model is better in comparison to others when it comes to fit observed count data set. Most recentlyRashid and Jan (2016) constructed a new lifetime distribution with real life application in environmental sciences

In this paper we propose a new count data model by compounding negative binomial distribution (NBD) with Kumaraswamy distribution (KSD) and some similarities of the proposed model will be shown with some already existing compound distribution.

2 Materials and Methods

A random variableis X said to have a negative binomial distribution (NBD) it its p.m.f is given by

$$f_1(x; r, p) = \binom{x+r-1}{x} p^r q^x; x = 0, 1, 2... \tag{1}$$

where $r > 0$ and $0 < p < 1$ are its parameters. The factorial moment of the NBD random variable X

$$\mu_{[k]} = \frac{\Gamma(r+k)}{\Gamma(r)} \left(\frac{q}{k}\right)^k \quad \text{for} \quad k = 1, 2, ... \tag{2}$$

$$E(X) = \frac{rq}{p} \quad \text{and} \quad V(X) = \frac{rq}{p^2}$$

A random variable X is said to have a Kumaraswamy distribution (KSD) if its pdf is given by

$$f_2(X;\alpha,\beta) = \alpha\beta x^{\alpha-1}\left(1-x^{\alpha}\right)^{\beta-1}, 0<x<1 \tag{3}$$

where $\alpha, \beta > 0$ are shape parameters. The raw moments of Kumaraswamy distribution are given by

$$\begin{aligned} E(X^r) &= \int_0^1 x^r f_2(X;\alpha,\beta)dx \\ &= \frac{\Gamma(\beta+1)\Gamma\left(1+\frac{r}{a}\right)}{\Gamma\left(1+\beta+\frac{r}{\alpha}\right)} \end{aligned} \tag{4}$$

Kumaraswamy distribution is a two parameter continuous probability distribution that has obtained by Kumaraswamy (1980) but unfortunately this distribution is not very popular among statisticians because researchers have not analyzed and investigated it systematically in much detail. Kumaraswmy distribution is similar to the beta distribution but unlike beta distribution it has a closed form of cumulative distribution function which makes it very simple to deal with. For more detailed properties one can see references Jones (2009)

Usually the parameters r and p in NBD are fixed constants but here we have considered a problem in which the probability parameter p is itself a random variable following KSD with p.d.f (3).

3 Definition of proposed model

If $X|p \sim NBD(r,p)$ where p is itself a random variable following Kumaraswamy distribution $KSD(\alpha,\beta)$ then determining the distribution that results from marginalizing over p will be known as a compound of negative binomial distribution with that of Kumaraswamy distribution which is denoted by $NBKSD(r,\alpha,\beta)$. It may be noted that proposed model will be a discrete since the parent distribution NBD is discrete.

Theorem 3.1: The probability mass function of a compound of $NBD(r,p)$ with $KSD(\alpha,\beta)$ is given by

$$f_{NBKSD}(X;r,\alpha,\beta) = \binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\frac{\Gamma(\beta+1)\Gamma\left(\frac{r+j}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)}$$

where $x = 0,1,2,...,r,\alpha,\beta > 0$

Proof: Using the definition (3), the p.m.f of a compound of $NBD(r,p)$with $KSD(\alpha,\beta)$ can be obtained as

$$\begin{aligned} f_{NBKSD}(X;r,\alpha,\beta) &= \int_0^1 f_1(x|P)f_2(p)dp \\ &= \alpha\beta\binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^j\int_0^1 p^{r+j+\alpha-1}(1-p^{\alpha})^{\beta-1}dp \end{aligned}$$

substituting $1-p^{\alpha}=z$, we get

$$\begin{aligned} f_{NBKSD}(X;r,\alpha,\beta) &= \beta\binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\int_{0}^{1} z^{\beta-1}(1-z)^{\frac{r+j}{\alpha}}dz \\ &= \beta\binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}B\left(\beta,\frac{r+j}{\alpha}+1\right) \\ &= \binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{r+j}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)} \qquad (5) \end{aligned}$$

where $x=0,1,2,...,r,\alpha,\beta>0$. From here a random Xvariable following a compound of NBD with KSD will be symbolized by NBKSD(r,α,β)

4 Reparameterization technique

There are very few continuous probability distributions in statistics whose support lies between 0 and 1 so in order to ascribe a suitable distribution to a probability parameter p we have a limited choice, to remove this limitation researchers try to reparameterize the probability parameter by equating p to $e^{-\lambda}$ where $\lambda>0$ is a random variable. So instead of ascribing a suitable probability distribution to parameter researchers ascribe a suitable distribution to the parameter λ by treating it as a random variable with support $\lambda>0$ and there are numerous probability distributions in statistics whose support lies $(0,\infty)$

In this section a similarity will be shown between a proposed model and a model which is obtained by compounding negative binomial distribution with generalized exponential distribution through reparameterization technique.

Proposition 4.1: The probability function of the proposed model gets coincide with the compound of NBD with GED obtained through reparameterization.

Proof: If $(X|\lambda)\sim NBD(r,p=e^{-\lambda}$ where λ is itself a random variable following a generalized exponential distribution (GED) with probability density function

$$f_3(\lambda;\beta,\alpha)=\beta\alpha(1-e^{\alpha\lambda})^{\beta-1}e^{-\alpha\lambda} \quad \text{for} \quad \alpha,\beta,\lambda>0 \qquad (6)$$

then determining the distribution that results from marginalizing over λ will give us a compound of NBD with GED

$$\begin{aligned} f_{NBGED}(x;\beta,\alpha) &= \int_{0}^{\infty} f_1(x|\lambda)f_3(\lambda,\beta,\alpha)d\lambda \\ &= \binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\int_{0}^{\infty} e^{-\lambda(r+j)}f_3(\lambda,\beta,\alpha)d\lambda \\ &= \binom{x+r-1}{x}\sum_{j=0}^{x}\binom{x}{j}(-1)^{j}\frac{\Gamma(\beta+1)\Gamma\left(\frac{r+j}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)} \qquad (7) \end{aligned}$$

where $x = 0, 1, 2, ..., r, \alpha, \beta > 0$. Interestingly, this gives rise to the same probability function as the probability function (5) of the proposed model. The probability function defined in (7) was obtained by Aryuyuen and Bodhisuwan (2013) hence our model can be treated as a simple and easy alternative since it has been obtained without reparameterization.

5 Nested distributions

In this particular section we show that the proposed model can be nested to different models under specific parameter setting.

Proposition 5.1: If $X \sim NBKSD(r, \alpha, \beta)$ then by setting $r = 1$ we get a compound of geometric distribution with Kummarswamy distribution.

Proof: For $r = 1$ in (1) NBD reduces to geometric distribution (GD) hence a compound of GD with KSD is followed from (5) by simply substituting $r = 1$ in it.

$$f_{GKSD}(X; \alpha, \beta) = \sum_{j=0}^{x} \binom{x}{j} (-1)^j \frac{\Gamma(\beta+1)\Gamma\left(\frac{j+1}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j+1}{\alpha}+1\right)} \quad \text{for} \quad x = 0, 1, 2, ..., r, \alpha, \beta > 0$$

which is the probability mass function of a compound of GD with KSD.

Proposition 5.2: If $X \sim NBKSD(r, \alpha, \beta)$ then by setting $\alpha = \beta = 1$ we obtain a compound of NBD distribution with uniform distribution.

Proof: For $\alpha = \beta = 1$ in KSD reduces to uniform (0,1) distribution therefore a compound NBD with uniform distribution is followed from (5) by simply putting $\alpha = \beta = 1$ in it.

$$f_{NBUD}(X; \alpha, \beta) = \binom{x+r-1}{x} \sum_{j=0}^{x} \binom{x}{j} \frac{(-1)^j}{r+j+1} \quad \text{for} \quad x = 0, 1, 2, ..., r, \alpha, \beta > 0$$

which is probability mass function of a compound of NBD with uniform distribution.

Proposition 5.3: If $X \sim NBKSD(r, \alpha, \beta)$ then by setting $\alpha = \beta = 1$ and we obtain a compound of geometric distribution with uniform distribution.

Proof: For $r = 1$ in (1), NBD reduces to geometric distribution and for $\alpha = \beta = 1$ Kumaraswamy distribution reduces to U(0,1) distribution hence a compound of geometric distribution with uniform distribution can be obtained from (5) by simply substituting $r = 1$ and $\alpha = \beta = 1$ in it.

$$f_{GUD}(X) = \sum_{j=0}^{x} \binom{x}{j} \frac{(-1)^j}{j+2} \quad \text{for} \quad x = 0, 1, 2, ...$$

which is the probability function of GD with U(0,1) distribution.

6 Mean and variance of the proposed model

In order to obtain the l^{th} moment of the proposed model $NBKSD(r,\alpha,\beta)$ about origin we need to apply the well-known results of probability theory viz

(i) Conditional expectation identity $E(X^l) = E_p(X^l|P)$ and

(ii) Conditional variance identity $V(X) = E_p(Var(X|P)) + Var_p(E(X|P))$

Since $X|P \sim NBD(r,p)$ where p is itself a random variable following $KSD(\alpha,\beta)$, therefore we have

$$\begin{aligned} E(X^l) &= E_p(X^l|P) \\ &= rE_p\left(\frac{1-p}{p}\right)^l \\ &= r\sum_{j=0}^{l}\binom{l}{j}(-1)^j\int_0^1 p^{j-1}f_2(p;\alpha,\beta)dp \end{aligned}$$

using the arguement (4) we get

$$E(X^l) = r\sum_{j=0}^{l}\binom{l}{j}(-1)^j\frac{\Gamma(\beta+1)\Gamma\left(\frac{j-l}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-l}{\alpha}+1\right)}$$

For $l=1$, we get the mean of NBKSD

$$E(X) = r\left(\frac{\Gamma(\beta+1)\Gamma(1-\frac{1}{\alpha})}{\Gamma(1+\beta-\frac{1}{\alpha})} - 1\right) = m_1$$

similarly we can find variance of NBKSD using conditional variance identity (ii)

$$\begin{aligned} V(X) &= E_p\left(\frac{r(1-p)}{p^2}\right) + Var_p\left(\frac{r(1-p)}{p}\right) \\ &= \frac{(r+r^2)\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+1\right)} - \frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\left(1+\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\right) \end{aligned}$$

7 Factorial moment of the proposed model

In this section we shall obtain factorial moment of the proposed model which is very helpful to study some of the important features such as mean, variance, standard deviation and so on.

Theorem 7.1 The factorial moment of order k of the proposed model is given by

$$\mu_{[k]}(X) = \frac{\Gamma(r+k)}{\Gamma(r)} \sum_{j=0}^{k} \binom{k}{j} (-1)^j \frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

where $X = 0, 1, 2, ..., r, \alpha\beta > 0$

Proof The factorial moment of order k of NBD is

$$m_k(X|p) = \frac{\Gamma(r+k)}{\Gamma(r)} \frac{(1-p)^k}{p^k}$$

since p itself is a random variable following KSD, therefore one obtains factorial moment of the proposed model by using the definition

$$\begin{aligned}
\mu_{[k]}(X) &= E_p(m_k(X|p)) \\
&= \frac{\Gamma(r+k)}{\Gamma(r)} E_p \left(\frac{1-p}{p}\right)^k \\
&= \frac{\Gamma(r+k)}{\Gamma(r)} \alpha\beta \sum_{j=0}^{k} (-1)^j \binom{k}{j} \int_0^1 p^{\alpha+j-k-1}(1-p^\alpha)^{\beta-1} dp
\end{aligned}$$

substituting $1 - p^\alpha = z$, we get

$$\begin{aligned}
&= \frac{\Gamma(r+k)}{\Gamma(r)} \beta \sum_{j=0}^{k} (-1)^j \binom{k}{j} \int_0^1 z^{\beta-1}(1-z)^{\frac{j-k}{\alpha}} dz \\
&= \frac{\Gamma(r+k)}{\Gamma(r)} \beta \sum_{j=0}^{k} (-1)^j \binom{k}{j} B\left(\beta, \frac{j-k}{\alpha}+1\right) \\
&= \frac{\Gamma(r+k)}{\Gamma(r)} \sum_{j=0}^{k} (-1)^j \binom{k}{j} \frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)} \qquad (8)
\end{aligned}$$

where $x = 0, 1, 2, ..., r, \alpha, \beta > 0$. For $k = 1$, we mean of NBKSD (r, α, β)

$$\mu_{[1]}(X) = r\left(\frac{\Gamma(\beta+1)\Gamma(1-\frac{1}{\alpha})}{\Gamma(1+\beta-\frac{1}{\alpha})} - 1\right) = m_1$$

$$\mu_{[2]}(X) = r(r+1)\left(\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+2\right)} - 2\frac{\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+2\right)} + \frac{1}{\beta+1}\right) = \nu_2$$

$$E(X^2) = \mu_{[2]}(X) + \mu_{[1]}(X) = \nu_2 + m_1 = m_2$$

similarly we can find third moment $E(X^3)$

$$V(X) = m_2 - m_1^2 = \frac{(r+r^2)\Gamma(\beta+1)\Gamma\left(1-\frac{2}{\alpha}\right)}{\Gamma\left(\beta-\frac{2}{\alpha}+1\right)} - \frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}$$

$$\left(1+\frac{r\Gamma(\beta+1)\Gamma\left(1-\frac{1}{\beta}\right)}{\Gamma\left(\beta-\frac{1}{\alpha}+1\right)}\right)$$

Standard deviation of NBKSD (r,α,β) model is $SD = \sqrt{V(X)}$

Corollary 7.2: The factorial moment of order k of a compound of NBD with uniform distribution is given by

$$\mu_{[k]}(X) = \frac{\Gamma(r+k)}{\Gamma(r)} \sum_{j=0}^{k} (-1)^j \left(\frac{\binom{k}{j}}{j-k+2}\right)$$

where $x = 0, 1, 2..., r, \alpha, \beta > 0$

Proof: Since KSD (α, β) reduces to Uniform distribution $(0,1)$ for $\alpha = \beta = 1$, therefore factorial moment of order k of a compound of NBD with uniform distribution $(0,1)$ can be obtained from (8) by simply substituting $\alpha = \beta = 1$ in it.

$$\mu_{[k]}(X) = \frac{\Gamma(r+k)}{\Gamma(r)} \sum_{j=0}^{k} (-1)^j \left(\frac{\binom{k}{j}}{j-k+2}\right)$$

Corollary 7.3: The factorial moment of a compound of GD with KSD is given by

$$\mu_{[k]}(X) = k \sum_{j=0}^{k} (-1)^j \binom{k}{j} \frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

where $x = 0, 1, 2..., r, \alpha, \beta > 0$

*Proof:*For $r = 1$ NBD reduces to GD and a factorial moment of order k of a compound of geometric distribution with KSD is obtained from (8) by putting $r = 1$ in it.

$$\mu_{[k]}(X) = k \sum_{j=0}^{k} (-1)^j \binom{k}{j} \frac{\Gamma(\beta+1)\Gamma\left(\frac{j-k}{\alpha}+1\right)}{\Gamma\left(\beta+\frac{j-k}{\alpha}+1\right)}$$

Corollary 7.4: The factorial moment of a compound of GD with uniform distribution is given by

$$\mu_{[k]}(X) = k \sum_{j=0}^{k} (-1)^j \left(\frac{\binom{k}{j}}{j-k+2}\right)$$

where $x = 0, 1, 2...,$

Prof: Substituting $r = 1$ and $\alpha = \beta = 1$ in (8) we get the result

8 Parameter estimation

In this section the estimation of parameters of NBKSD (r, α, β) model will be discussed through method of moments and maximum likelihood estimation.

8.1 *Method of moments*

In order estimate three unknown parameters of NBKSD (r, α, β) model by the method of moments we need to equate first three sample moments with their corresponding population moments.

$$m_1 = \gamma_1;\ m_2 = \gamma_2 \ and \ m_3 = \gamma_3$$

where $\gamma_i = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the ith sample moment and is the ith corresponding population moment and the solution for $r,\ \alpha \ and \ \beta$ and may be obtained by solving above equations simultaneously.

8.2 *Maximum likelihood estimation*

The estimation of parameters of NBKSD (r, α, β) model via maximum likelihood estimation method requires the log likelihood function NBKSD (r, α, β) of

$$\begin{aligned} \pounds(X; r, \alpha, \beta) &= \log L(X; r, \alpha, \beta) = \sum_{i=1}^{n} log \binom{x_i + r - 1}{x_i} + \beta^n \\ &+ \sum_{i=1}^{n} log \left(\sum_{j=0}^{x} \binom{x}{j} (-1)^j B\left(\beta, \frac{j + r}{\alpha} + 1\right)\right) \end{aligned} \tag{9}$$

The maximum likelihood estimate of $\Theta = (r, \alpha, \beta)$ can be obtained by differentiating (9) with respect unknown parameters $r,\ \alpha \ and \ \beta$ respectively and then equating them to zero.

$$\frac{\partial}{\partial r}\pounds(X; r, \alpha, \beta) = \sum_{i=1}^{n} \psi(x_i + r) - n\psi(r) + \sum_{i=1}^{n} \left(\frac{\sum_{j=0}^{x_i} \binom{x}{j} (-1)^j \frac{\partial}{\partial r} \left(\frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)} \right)}{\sum_{j=0}^{x_i} \binom{x}{j} (-1)^j \frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)}} \right)$$

$$\frac{\partial}{\partial \alpha}\pounds(X; r, \alpha, \beta) = \sum_{i=1}^{n} \left(\frac{\sum_{j=0}^{x_i} \binom{x}{j} (-1)^j \frac{\partial}{\partial \alpha} \left(\frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)} \right)}{\sum_{j=0}^{x_i} \binom{x}{j} (-1)^j \frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)}} \right)$$

$$\frac{\partial}{\partial\beta}\pounds(X;r,\alpha,\beta)=\sum_{i=1}^{n}\left(\frac{\sum_{j=0}^{x_i}\binom{x}{j}(-1)^j\frac{\partial}{\partial\beta}\left(\frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)\Gamma(\beta+1)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)}\right)}{\sum_{j=0}^{x_i}\binom{x}{j}(-1)^j\frac{\Gamma\left(1+\frac{r+j}{\alpha}\right)\Gamma(\beta+1)}{\Gamma\left(\beta+\frac{r+j}{\alpha}+1\right)}}\right)$$

These three derivative equations cannot be solved analytically, therefore $\hat{r},\hat{\alpha},\ and\ \hat{\beta}$ will be obtained by maximizing the log likelihood function numerically using Newton-Raphson method which is a powerful technique for solving equations iteratively and numerically.

9 Applications of the proposed NBKSD model

Classical Poisson distribution plays an important role in modeling count data processes but it requires a strong condition of independent successive events which may not be the characteristic of the count data under consideration such as a data given in table (1-3). Furthermore the equality of mean and variance of the observed counts is hardly satisfied in practice, negative binomial distribution can be used in such cases but negative binomial distribution is better for over dispersed count data that are not necessarily heavy tailed in such situations compound distribution models serves efficiently well.

9.1 Application in Genetics

Genetics is the science which deals with the mechanisms responsible for similarities and differences among closely related species. The term genetic is derived from the Greek word genesis meaning grow into or to become. So, genetic is the study of heredity and hereditary variations it is the study of transmission of body features that is similarities and difference, from parents to offsprings and the laws related to this transmission, any difference between individual organisms or groups of organisms of any species, caused either by genetic difference or by the effect of environmental factors, is called variation. Variation can be shown in physical appearance, metabolism, behavior, learning and mental ability, and other obvious characters. In this section the potentiality of proposed model will be justified by fitting it to the reported genetics count data sets of Catcheside et al.

9.2 Application in ecology

The relationship between organisms and their environment can be studied through ecology, a branch of biology. Many models have been used to fit ecological data, in this particular section an attempt has been made to fit ecological count data on haemocytometer yeast cell counts per square, on European red mites on apple leaves by proposed NBKSD model.

Table 1: Distribution of number of Chromatid aberrations (0.2 g chinon 1, 24 hours)

Number of abberations	Observer frequency	PD	NBD	NBKSD
0	268	231.5	279.1	269.8
1	87	126.1	71.1	82.5
2	26	34.7	27.5	27.8
3	9	6.3	11.8	10.6
4	4	0.8	5.3	4.5
5	2	0.1	2.5	2.1
6	1	0.1	1.1	1.6
7+	3	0.1	0.5	0.5
Total	400	400	400	400
Parameter estimate(s)		$\hat{\theta} = 0.545$	$\hat{r} = 0.49$ $\hat{p} = 0.48$	$\hat{r} = 1.74$ $\hat{\alpha} = 5.94$ $\hat{\beta} = 2.0$
Chi-square estimate		38.20	4.78	0.77
DF		2	2	1
P value		0.00	0.09	0.37

Table 2: Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383), Exposure -60 g/kg

Class/Exposure(g/kg)	Observer frequency	PD	NBD	NBKSD
0	413	374.0	407.6	411.3
1	124	177.4	127.6	129.4
2	42	42.1	42.9	39.3
3	15	6.6	14.7	12.9
4	5	0.8	5.1	4.6
5	0	0.1	1.8	1.8
6	2	0.0	0.6	0.7
Total	601	601	601	601
Parameter estimate(s)		$\hat{\theta} = 0.47$	$\hat{r} = 0.87$ $\hat{p} = 0.64$	$\hat{r} = 2.11$ $\hat{\alpha} = 9.32$ $\hat{\beta} = 2.92$
Chi-square estimate		48.16	0.23	0.73
DF		2	2	1
P value		0.000	0.89	0.39

Table 3: Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin (NSC-45383) Exposure -70 g/kg

Class/Exposure(g/kg)	Observer frequency	PD	NBD	NBKSD
0	200	172.5	205.5	199.3
1	57	95.4	60.8	62.4
2	30	26.4	21.1	22.8
3	7	4.9	7.7	8.9
4	4	0.7	2.8	3.6
5	0	0.1	1.0	1.5
6	2	0.0	0.4	0.6
Total	300	300	300	300
Parameter estimate(s)		$\hat{\theta} = 0.55$	$\hat{r} = 0.74$ $\hat{p} = 0.60$	$\hat{r} = 16.63$ $\hat{\alpha} = 26.71$ $\hat{\beta} = 0.79$
Chi-square estimate		29.6	4.24	3.15
DF		2	1	1
P value		0.001	0.039	0.075

Table 4: Mammalian cytogenetic dosimetry lesions in rabbit lymphoblast induced by streptonigrin

Number of red mites per leaf	Observer frequency	PD	NBD	NBKSD
0	38	25.3	35.0	36.3
1	17	29.1	21.8	20.4
2	10	16.7	11.9	10.8
3	9	6.4	6.5	5.7
4	3	1.8	2.8	3.04
5	2	0.4	1.2	1.62
6	1	0.2	0.5	0.87
7+	0	0.1	0.2	0.47
Total	80	80	80	80
Parameter estimate(s)		$\hat{\theta} = 1.15$	$\hat{r} = 1.35$ $\hat{p} = 0.54$	$\hat{r} = 34.61$ $\hat{\alpha} = 33.45$ $\hat{\beta} = 1.16$
Chi-square estimate		18.27	2.90	2.55
DF		2	1	1
P value		0.001	0.08	0.10

Figure 1: Graphs of proposed NBKSD model

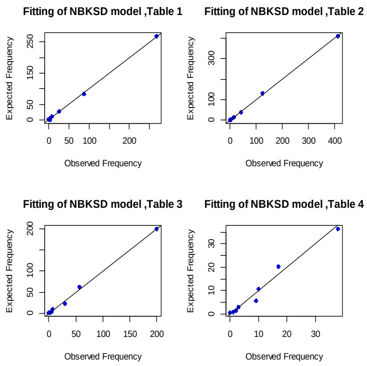
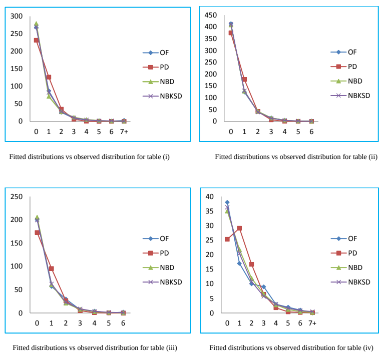


Figure 2: Graphs of fitted PD, NBD and NBKSD to data sets given in table 1:4



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